

※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

1 至 2 題為多選題，答案可能不只一個，每題皆須列出計算過程。

1. (15%) The four eigenvalues of a  $4 \times 4$  matrix  $A$  are 1, 1, 2, and 3. Then (從下列選項中選出正確的敘述，須列出計算過程或說明理由)  
(A) the determinant of  $A$  is 6.  
(B) the trace of  $A$  is 7.  
(C)  $\text{rank}(A - I) = 3$ , where  $I$  is the  $4 \times 4$  identity matrix.  
(D) the determinant of the adjoint matrix  $A^+$  is 6.  
(E)  $A$  is a simple matrix.
  
2. (15%) Given  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ .  $A = QR$  is the Q-R factorization (orthogonal- triangular decomposition) of  $A$ . Then (從下列選項中選出正確的敘述，須列出計算過程或說明理由)  
(A) the two eigenvalues of  $A$  are both 1.  
(B) the eigenvectors of  $A$  are  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ .  
(C)  $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ .  
(D)  $R = A$ .  
(E)  $RQ = A$ .
  
3. (20%)  $S$  is a portion of the surface  $x + y^2 - z = 0$ , and its projection on the  $xy$  plane is the region enclosed by  $(x-1)^2 + (y-1)^2 = 1$ . Consider a vector field  $\mathbf{u} = y^2\mathbf{i} + xy\mathbf{j} + (z-x)\mathbf{k}$ . Compute the value of the surface integral  $\int_S \mathbf{u} \cdot d\boldsymbol{\sigma}$ .
  
4.  $y = y(t)$  is a function of  $t$ . Solve the following equations:
  - (a) (15%)  $y' - y = e^t$ .
  - (b) (15%)  $y'' - 2\frac{y'}{t} + 2\frac{y}{t^2} - t = 0$ .
  - (c) (20%)  $2ty'' + y' + 2y = 0$ .