1. Laser cooling is a modern technique to cool atoms to extremely low temperature. The key point is to tune the laser frequency slightly lower than the nominal frequency of an electronic transition in the atom. Derive the Doppler shift using Lorentz transformation in one dimension (8%). From the above derivation, explain why the atom can absorb the laser light (2%). Describe the phenomenon of spontaneous emission (3%). Explicitly show that after the spontaneous emission, the atom is cooled in the Lab frame (10%). What is the condition that the atom can keep its original energy (2%)?

2. Blackbody radiation plays an important role in the development of quantum theory. What is the definition of blackbody and how to prepare such an ideal blackbody experimentally (5%)? Have a rough sketch for the energy density of a blackbody cavity as a function of wavelength for three different temperatures, 100K, 200K, and 300K, respectively. There is no need to mark the absolute scale and these three curves should be overlaid on the same graph (5%). Derive the blackbody radiation spectrum based on Planck's approach. First you need to determine the number of states with similar frequencies in a cubic cave (5%). Then you need to calculate the average energy for each mode (i.e. with fixed frequency) assuming that it is in thermal equilibrium with the simple harmonic oscillator on the wall of the cave (8%). Note that the energies of the S.H.O. are quantized according to Planck's postulate. You should write his postulate down explicitly (2%). The Boltzmann distribution has the following form: $\frac{1}{kT}e^{-\epsilon/kT}$, where ϵ represents the state energy, k is the Boltzmann constant and T is the absolute temperature.

*** Following (no. 3-12) are multiple-choice problems, which may have more than one correct choice.*

3. The fine-structure splittings of the energy levels of the hydrogen atom (5%)

   (a) can be explained by the Bohr model, using just the Bohr quantization rules;

   (b) are of the order of the fine-structure constant $\alpha = e^2/(4\pi\epsilon_0\hbar c) \simeq 1/137$, compared to the original energy level spacings;

   (c) leave the two levels $^2S_{1/2}$ and $^2P_{1/2}$ (of principal quantum number $n = 2$) degenerate;

   (d) are due solely to the spin-orbit coupling of the electron.

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4. Which of the following physical quantities can be measured simultaneously with p_z, the z-component of the momentum, to any desired accuracy? (5%)

   (a) l_x, the x-component of the orbital angular momentum;

   (b) l_z, the z-component of the orbital angular momentum;

   (c) s_z, the z-component of the spin angular momentum;

   (d) The potential energy $V(\vec{r})$.

5. With a_0 denoting the Bohr radius, the expectation value of $\frac{1}{r}$ of an electron in the $2s$ state of a hydrogen atom is equal to (5%)

   (a) $\frac{1}{a_0}$;

   (b) $\frac{1}{2a_0}$;

   (c) $\frac{1}{4a_0}$;

   (d) $\frac{1}{8a_0}$.

6. Consider three systems of noninteracting identical particles which are bosons, fermions, and classical particles, respectively. For the same density, energy and temperature, which system tends to have the most particles in the same state and which system has the least? (5%)

   (a) Bosons most, fermions least;

   (b) Classical particles most, fermions least;

   (c) Classical particles most, bosons least;

   (d) Fermions most, classical particles least.

7. Of the four possible transitions for a hydrogen atom listed below, which transition emits the photons having the shortest wavelength? Here, n_i and n_f denote the principal quantum numbers of the initial and final states, respectively. (5%)

   (a) $n_i = 2 \rightarrow n_f = 5$;

   (b) $n_i = 5 \rightarrow n_f = 3$;

   (c) $n_i = 7 \rightarrow n_f = 4$;

   (d) $n_i = 4 \rightarrow n_f = 7$.

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8. Which of the following states (given in the spectroscopic notation ^{2s+1}L) cannot be formed by two p-electrons in an atom? (5%)

   (a) ^3S;
   (b) ^1P;
   (c) ^1D;
   (d) ^1S.

9. Which of the following is the selection rule for the emission of a photon by an atomic electron? (5%)

   (a) $\Delta l = \pm 1$, $\Delta j = \pm 1$;
   (b) $\Delta l = \pm 1$, $\Delta m = \pm 1$;
   (c) $\Delta n = \pm 1$, $\Delta l = \pm 1$;
   (d) $\Delta l = \pm 1$, $\Delta j = 0, \pm 1$.

10. Which of the following is not related to the particle-wave duality? (5%)

   (a) Photoelectric effect;
   (b) Stern-Gerlach experiment;
   (c) Compton effect
   (d) Thompson's electron diffraction experiment.

11. A pion π^- is captured by a deuteron d in an S-orbit to produce two neutrons $2n$ in the nuclear reaction $\pi^- + d \to 2n$. The spin of the pion is zero and that of the deuteron is 1. On the other hand, the pion has a negative parity and the deuteron has a positive parity. Since the nuclear reaction conserves both the total angular momentum and parity, the two neutrons must be in (5%)

   (a) a singlet state with relative orbital angular momentum $l = 0$;
   (b) a singlet state with relative orbital angular momentum $l = 1$;
   (c) a triplet state with relative orbital angular momentum $l = 0$;
   (d) a triplet state with relative orbital angular momentum $l = 1$.

12. Which of the following functions are eigenfunctions of the momentum operator p_x? (5%)

   (a) $A\sin(kx)$;
   (b) $A\sin(kx) - iA\cos(kx)$;
   (c) $A\sin(kx) + B\cos(kx)$;
   (d) $Ae^{ik(x-a)}$.