1. (20%) Explain the following terms briefly. (a) A stable algorithm. (b) Quadratic convergence. (c) Relative error. (d) Truncation error. (e) Preconditioning

2. (8%) (a) Let $A \in R^{n\times n}$ be given. Assume that (i) A has n linearly independent eigenvectors, x_k, for k=1,…,n. (ii)The eigenvalues λ_k satisfy $|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \cdots \geq |\lambda_n|$ (iii) The vector $z \in R^n$ is such that $z = \sum_{k=1}^{n} \xi_k x_k$ and $\xi_1 \neq 0$. Show that (i) $\lim_{N\to\infty} \frac{A^N z}{\lambda_1^N} = cx_1$ for some $c \neq 0$ and (ii) $\lim_{N\to\infty} \frac{z^T A^N z}{z^T A^{N-1} z} = c\lambda_1$. Here z^T is the transpose of z.
(7%) (b) Write a pseudo-code that uses the results in (a) to compute the dominant eigenvalue of a matrix A.
(5%) (c) What is the complexity of your method in (b)?
(5%) (d) What is the convergence rate of your method in (b)?

3. Consider the following pseudo-code.

```
input x, t, m
Step (1) set n=1
   y=x-1, SUM=0; PWR=y; TRM=y; SGN=-1;
Step (2) while (n ≤ m) do
   Step (3) SGN = -SGN; SUM = SUM + SGN*TRM; PWR = PRW*y; TRM = PWR/(n+1);
   Step (4) if (abs(TRM) < t) then output(n); stop.
   Step (5) set n = n + 1;
Step (6) output ('Method Failed'); stop.
```

(5%) (a) What are the meaning of the input variable x, t, and m?
(5%) (b) What does this algorithm do?
(5%) (c) What are the stopping criteria of this algorithm? Why are the criteria chosen?
(5%) (d) Compute the floating point operation counts of the algorithm.

4. Consider the initial value problem (IVP) $y' = -y \ln y$, for $y(0) = y_0$.
(5%) (a) Derive Euler's method and the trapezoid method to solve the IVP.
(5%) (b) Derive a predictor-corrector method to solve the IVP by using the Euler's method and the trapezoid method.
(5%) (c) In stead of using the predictor-corrector method, explain how you can use Newton's method to precede the trapezoid method described in (b).
(5%) (d) Write a pseudo-code to implement the predictor-corrector method in (b).

5. (15%) Derive a method to compute multiple roots of a one-dimensional function. Describe your method in detail. Can you guarantee that you will find all the roots?

試題隨卷繳回