國立臺灣大學98學年度碩士班招生考試試題

(1) (20 pts) Let p be a prime number. We consider the polynomial $f(x) = x^{p-1} + x^{p-2} + \ldots + 1$.
 (a) Prove that $f(x)$ is irreducible in $\mathbb{Q}[x]$.
 (b) Factorize $f(x)$ in $\mathbb{F}_p[x]$, where $\mathbb{F}_p$ denotes the finite field of p elements.

(2) (15 pts) Let G, H be finite abelian groups. Prove that $G \times G$ and $H \times H$ are isomorphic if and only if G and H are isomorphic.

(3) (15 pts) Consider 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ over $\mathbb{R}$. Suppose that $0 \leq a, b, c, d \leq 1$. Determine the least upper bound of the eigenvalues among all matrices of the given form.

(4) (15 pts) Let $\pi : G \to G'$ be a homomorphism with kernel K. Let $H < G$ be any subgroup. Prove that $\pi^{-1}\pi(H) = HK = KH$.

(5) (15 pts) Find all integers satisfying $x^2 - x - 2 \equiv 0 \pmod{25}$.

(6) (20 pts) Construct a finite field of 343 elements. Verify your construction.

試題隨卷繳回