1. **(8 points)** Prove that if $P(A)>0$ and $P(B)>0$, then:
   A. If A and B are mutually exclusive, they cannot be independent. (4 points)
   B. If A and B are independent, they cannot be mutually exclusive. (4 points)

2. **(15 points)** 9 data points are randomly drawn from a normal distribution with a known standard deviation of 2. Suppose you want to test the null hypothesis H_0: $\mu = 7$ against the alternative hypothesis H_1: $\mu \neq 7$ at the 5% level of significance. Suppose the 9 data points have a sample mean of 7.5.
   A. Calculate the test statistic. What is the rejection region for the test statistic? (4 points)
   B. What is the p-value for these data? What is your conclusion regarding the null hypothesis? (4 points)
   C. Determine the power of the test using the alternative hypothesis H_1: $\mu = 7.2$. (7 points)

3. **(27 points)** A health economist is interested in the effect of wealth on mental health. 5 people were randomly selected, and information on their monthly income and happiness score (a higher score indicates greater happiness) were collected:

ID	1	2	3	4	5
Income	23	21	18	17	21
Happiness score	10	9	5	4	7

   A. Calculate the least squares line of regression of happiness score on income. (6 points)
   B. Interpret (i) the constant term and (ii) the estimated coefficient of income. (4 points)
   C. Test the significance of the estimated coefficient of income at the 5% significance level, given that the standard error of the coefficient estimate is 0.1667. Clearly write down each step of the hypothesis testing procedure. (6 points)
   D. Calculate the 95% confidence interval for the estimated coefficient of income. (4 points)
   E. Given the following table of analysis of variance for the regression, calculate the coefficient of determination (R^2) and interpret it. (3 points)

Source of variation	Degrees of Freedom	Sum of Squares	Mean Squares	F
Regression	1	24	24	36
Residual	3	2	0.6667	
Total	4	26		

   F. What is the predicted happiness score for a person with an income of 21.3? (2 points)
   G. Is it appropriate to predict the happiness score for a person with an income of 47.3 using the estimated regression equation? Explain your answer. (2 points)

見背面

4. **(8 points)** Die A has orange on two faces and blue on four faces, Die B has orange on three faces and blue on three faces, Die C has orange on four faces and blue on two faces. All are fair dice. If the three dice are rolled, find the probability that exactly two of the three dice come up orange.

5. **(15 points)** Let X have a uniform distribution on the interval (0, 1). Given that $X = x$, let Y have a uniform distribution on the interval $(0, e^x)$.

   (a) Find the joint pdf of X and Y. Sketch the region where $f(x, y) > 0$. (5 points)
   (b) Find $E(Y \mid x)$, the conditional mean of Y, given that $X = x$. Draw this line on the region sketched in part (a). (5 points)
   (c) Find $f_Y(y)$, the marginal pdf of Y. Be sure to include the domain. (5 points)

6. **(12 points)** Let the following numbers represent the order statistics of the $n = 27$ observations obtained in a random sample from a certain population of monthly earnings in Taiwan (measured in hundreds of NT dollars):

261	269	271	274	279	280	283	284	286	287	292	293	296
300	304	305	313	321	322	329	341	343	356	364	391	417
476												

   (a) Give point estimates of $\pi_{0.25}, median\ (\pi_{0.5}), \pi_{0.75}$. (6 points)

   (b) The interval (y_4, y_{10}) could serve as a confidence interval for $\pi_{0.25}$, that is $1-\alpha = P(y_4 < \pi_{0.25} < y_{10})$. Use the normal approximation to find the confidence coefficient $1-\alpha$. (6 points)

7. **(15 points)** A random sample $X_1, X_2, \ldots, X_n$ of size n is taken from a Poisson distribution with a mean of λ, $0 < \lambda < \infty$. The probability mass function of X is $f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$, $x = 0, 1, 2.....$

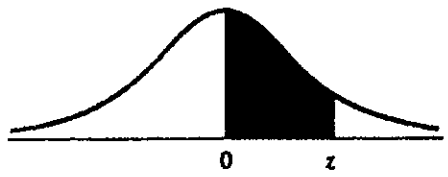
   (a) Show that the maximum likelihood estimator for λ is $\hat{\lambda} = \bar{X}$.(4 points)

   (b) Show that the method-of-moments estimator for λ is $\tilde{\lambda} = \bar{X}$.(3 points)

   (c) Show that $\bar{X}$ is an unbiased estimator of λ.(4 points)
   (d) Let X equal the number of flaws per 100 feet of a used computer tape. Assume that X has a Poisson distribution with a mean of λ. If 40 observations of X yielded 3 zeros, 8 ones, 10 twos, 6 threes, 7 fours, 4 five, and 2 six, find the maximum likelihood estimate of λ. (4 points)

接次頁

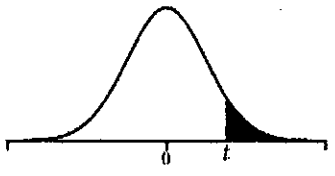
Table1. The probability of standard normal distribution: Pr(0<Z<z)



Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890

Table 2. Critical values for student's t-distribution.

Critical Values for Student's t-Distribution.



	Upper Tail Probability: Pr($T > t$)									
df	0.2	0.1	0.05	0.04	0.03	0.025	0.02	0.01	0.005	0.0005
1	1.376	3.078	6.314	7.916	10.579	12.706	15.895	31.821	63.657	636.619
2	1.061	1.886	2.920	3.320	3.896	4.303	4.849	6.965	9.925	31.599
3	0.978	1.638	2.353	2.605	2.951	3.182	3.482	4.541	5.841	12.924
4	0.941	1.533	2.132	2.333	2.601	2.776	2.999	3.747	4.604	8.610
5	0.920	1.476	2.015	2.191	2.422	2.571	2.757	3.365	4.032	6.869
6	0.906	1.440	1.943	2.104	2.313	2.447	2.612	3.143	3.707	5.959
7	0.896	1.415	1.895	2.046	2.241	2.365	2.517	2.998	3.499	5.408
8	0.889	1.397	1.860	2.004	2.189	2.306	2.449	2.896	3.355	5.041
9	0.883	1.383	1.833	1.973	2.150	2.262	2.398	2.821	3.250	4.781
10	0.879	1.372	1.812	1.948	2.120	2.228	2.359	2.764	3.169	4.587

試題隨卷繳回