1. (15 points) Consider the sequence of functions $\{f_n\}_{n=1}^{\infty}$, where $f_n(x) = \frac{x}{1+nx^2}$. Does f_n converge uniformly on $\mathbb{R}$? Justify your result.

2. (15 points) Show that $\sum_{m,n=1}^{\infty}(m+n)^{-p}$ converges if and only if $p > 2$.

3. (20 points) Let $f(x,y) : \mathbb{R}^2 \to \mathbb{R}$ be a C^1 function. Show that we can find two points $p, q \in \mathbb{R}^2$ so that $p \neq q$ and $f(p) = f(q)$.

4. (15 points) Assume $\{a_n\}_{n=1}^{\infty}$ are positive, and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$. Show that $\lim_{n\to\infty}(a_n)^{1/n} = L$.

5. (15 points) Let S be the surface formed by paraboloid $z = 1 - x^2 - y^2, z \geq 0$, and the unit disk centered at the origin in xy plane. Let $F = (0,0,z)$. Compute $\int\int_S F \cdot n ds$, where n is the unit outward normal vector on S.

6. (20 points) Consider the integral $g(x) = \int_0^{\infty} \frac{\sin t}{t} e^{-tx} dt$. Show that the integral converges uniformly for $x \geq 0$.

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