Any device with computer algebra system is prohibited during the exam.

**Section A. Fill in the blanks.**

- Only answers will be graded.
- Label clearly your answer to each blank *with the number of each blank* on the answer sheet.
- 5 points are assigned to each blank.

1. (a) $\lim_{x\to 0} \dfrac{5\cot(x) + 6\sin\frac{1}{x}}{7\csc(x) - 8\sin\frac{1}{x}} = \boxed{(1)}$

   (b) $\lim_{x\to 0} \left(e^{2x} - 2x - 2x^2\right)^{\frac{1}{x^3}} = \boxed{(2)}$

   (c) $\lim_{n\to\infty} \sum_{k=1}^{n} \dfrac{kn + n^2}{(k^2 + kn + n^2)^{\frac{3}{2}}} = \boxed{(3)}$

2. (a) Let $f(x,y) = (x+2)^{y+2}$. Then $\dfrac{d^2}{dt^2} f(t,t^2)\Big|_{t=0} = \boxed{(4)}$.

   (b) Let $g(x) = \dfrac{1}{\sqrt{x^2+2x+5}}$. Then $g^{(6)}(-1) = \boxed{(5)}$.

3. Consider a function $f : (-\pi,\pi) \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \sin x + \dfrac{ax}{\sin x} & \text{if } -\pi < x < 0 \\ x^2 + bx + 5 & \text{if } 0 \le x < \pi \end{cases}$$

   If $f$ is differentiable on $(-\pi,\pi)$, then $(a,b) = \boxed{(6)}$.

4. Consider the parametric curve $x = 2t^2 + 1$, $y = 4t$. Let $P$ be the point $(2p^2+1, 4p)$. The greatest value of $p$ such that the normal to the curve at $P$ passes through $(31,-24)$ is $p = \boxed{(7)}$.

5. Let $f(x,y,z) = \displaystyle\int_z^{x-y^2} \frac{e^{t^2}}{t^2+4}\,dt$. The linearization of $f(x,y,z)$ at $(1,1,0)$ is $L(x,y,z) = \boxed{(8)}$.

6. (a) $\displaystyle\int_0^1 x(\sin x + \sin^{-1} x)\,dx = \boxed{(9)}$.

   (b) Let $D$ be the region enclosed by the curve $y = (10x - x^2 - 21)^{\frac{1}{4}}$ and the $x$-axis. The volume of the solid obtained by revolving $D$ about the $x$-axis is $\boxed{(10)}$.

7. Let $f(x,y) = 2x^3 - 12xy + y^3 + 13$. Let $P = (p,q)$ be the point on $\mathbb{R}^2$ at which the rate of change of $f(x,y)$ in the direction $\mathbf{i}+\mathbf{j}$ is the smallest. Then $(p,q) = \boxed{(11)}$.

8. (a) $\displaystyle\int_1^e \int_{\ln x}^1 \frac{1}{(e^y - y)^2}\,dy\,dx = \boxed{(12)}$.

   (b) Let $R = \{(x,y) \in \mathbb{R}^2 : 1 \le x^2 + y^2 \le 4,\ x \ge 0,\ 0 \le y \le 1\}$. Then $\displaystyle\iint_R \frac{xy}{x^2+y^2}\,dA = \boxed{(13)}$.

9. (a) The work done by the force field $\mathbf{F}(x,y) = (\sqrt{1+x^3})\,\mathbf{i} + (xy)\,\mathbf{j}$ in moving a particle along a triangular path with vertices $(0,0)$, $(1,0)$, $(2,2)$ counter-clockwisely is $\boxed{(14)}$.

   (b) Let $S$ be part of the cone $z = \sqrt{2x^2+2y^2}$ that lies below the plane $x + z = 1$. Then $\displaystyle\iint_S x\,dS = \boxed{(15)}$.

   (c) Let $D$ be a closed surface in $\mathbb{R}^3$, oriented outward. The maximum flux of the vector field

$$\mathbf{F}(x,y,z) = (x + 2x^3z)\mathbf{i} - y(x^2+z^2)\mathbf{j} - (3x^2z^2 + 4y^2z)\mathbf{k}$$

   among all possible choices of $D$ is $\boxed{(16)}$.

10. The greatest value of $p$ such that the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot \tan\left(\frac{1}{\sqrt{n^p}}\right) \cdot \ln\left(1 + \frac{1}{n^{2p}}\right)$ converges conditionally is $p = \boxed{(17)}$.

題號： 17



**Section B. Long Question.**

- Solve the following problem. You need to write down a complete and correct argument to receive full credits.
- Your work is graded on the quality of your writing as well as the validity of the mathematics.
- 15 points are assigned to this question.

1. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{x \cdot \sin(y^2)}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$

(a) Is $f$ continuous at $(0,0)$? Justify your answer.

(b) Let $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ be a unit vector. Find the directional derivative of $f$ at $(0,0)$ in the direction $\mathbf{u}$. Express your answer in terms of $a$ and $b$.

(c) Find the direction(s) that $f$ changes the most rapidly at $(0,0)$.

**試題隨卷繳回**