

第一大題選擇題，考生應作答於「答案卡」。

第一大題 (共 30 分) 單選題

請在答案卡上塗上你的答案。每一個選擇題的答案只有一個。共十題，每題 3 分。

A consumer lives for two periods, current period t and future period $t+1$. To maximize his lifetime utility, he solves the following optimal consumption-saving problem.

$$\max_{C_t, C_{t+1}} u(C_t) + \beta u(C_{t+1})$$

Subject to the lifetime budget constraint.

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t},$$

where r_t is the interest rate, Y_t is his income at t , and Y_{t+1} is income at $t+1$.

Answer the following 10 multiple-choices questions using the above two-periods optimal consumption framework. In the following, MPC refers to the marginal propensity to consume.

- One of the implications of the intertemporal budget constraint is that:
 - current consumption does not have to equal current income
 - lifetime consumption equals total wealth
 - there can be income transfers between today and the future
 - All of the above answers are correct.
 - None of these answers is correct.
- In the optimal consumption model, if $\beta(1+r) < 1$, then which of the followings is correct?
 - current consumption is larger than future consumption
 - the current consumption is equal to the permanent income
 - the current consumption is less than the permanent income
 - none of the above are correct
- An increase in the interest rate will:
 - Cause current consumption to increase
 - Have negative income effect on C_t of savers
 - Have negative total effect on C_{t+1} of savers
 - Have negative income effect on borrowers
- Which of the following scenarios will make the MPC equal to 1?
 - A transitory income increases at t to a saver
 - A permanent income increases at both t and $t+1$
 - An expected income changes at $t+1$
 - A transitory income increases at t to a borrower
- If the interest rate r_t decreases, the substitution effect suggests that
 - C_t will increase and C_{t+1} will decrease.
 - C_t will decrease and C_{t+1} will increase.

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- (C) both C_t and C_{t+1} will increase.
(D) both C_t and C_{t+1} will decrease.
6. Suppose you are a saver. When the real interest rate r_t decreases, suppose that the income effect is stronger than the substitution effect, what will happen to C_t and C_{t+1} ?
- (A) C_t will increase and C_{t+1} will decrease
(B) C_t will decrease and C_{t+1} will increase
(C) both C_t and C_{t+1} will increase.
(D) both C_t and C_{t+1} will decrease
7. Consider the case of log utility for both periods, that is, $u(C_t) = \ln C_t$. If the change rate of consumption is 4% and suppose $\beta = 0.98$. In order for the Euler equation to hold in this case, what value must the real interest rate take? (Round off your answer and select the best fit option.)
- (A) 5%
(B) 6%
(C) 7%
(D) 8%
8. What is the impact on current consumption when the real interest rate increases:
- (A) C_t decreases due to the negative income effect
(B) C_t increases due to the positive income effect
(C) C_t decreases due to the intertemporal substitution effect
(D) C_t increases due to the intertemporal substitution effect
9. Which of the following statements of the Euler equation is false? If you think every statement is correct, select (E).
- (A) It equates the marginal cost and benefits of saving
(B) It equates the marginal utility of consumption today and the marginal utility of consumption tomorrow
(C) It implies that the marginal rate of substitution between consumption today and tomorrow is $(1+r)$
(D) When the Euler equation holds with equality, it implies that the MPC is less than 1
(E) All the above statements are correct, none of them are false
10. Suppose $\beta(1+r) < 1$ and the consumer is financially constrained. That is, he would like to borrow more but could not due to the borrowing constraint. Then which is correct:
- (A) His MPC is less than 1
(B) The marginal utility of current consumption is smaller than the marginal utility of future consumption
(C) The marginal utility of current consumption is larger than the marginal utility of future consumption
(D) An increase in the interest rate will result in a positive income effect for him

第二大題 (共 20 分) Short Answer Questions

※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

The consumption-leisure problem: now we consider a static problem. The consumer chose his optimal labor supply and leisure:

$$\max_{c,l} u(c,l) = \ln c - \frac{\gamma\epsilon}{1+\epsilon} n^{\frac{1+\epsilon}{\epsilon}}$$

Subject to the budget constraint

$$c = w \cdot n$$

where c denotes consumption, n labor supply, and l leisure. $n+l=l$. w is the wage. $\gamma > 0$ is the degree of disutility from work, and ϵ is the parameter.

- A. (6 points) Form the Lagrangian of this problem and solve for the optimal conditions with respect to consumption and with respect to labor. Clearly write down the definition of the marginal rate of substitution between leisure and consumption.
- B. (6 points) Suppose $\epsilon = 1$, and $\gamma > 0$. Solve for the optimal consumption and labor supply in question A. Write clearly on how the value of γ affects the answer. Explain how wage affects the optimal labor supply and why.
- C. (8 points) Suppose now the utility function follows the following GHH preference:

$$\max_{c,l} u(c,l) = \frac{1}{1-\phi} \left(c - \frac{\gamma\epsilon}{1+\epsilon} n^{\frac{1+\epsilon}{\epsilon}} \right)^{1-\phi}$$

Explain the unique feature of this GHH utility function on labor decision. Explain how wage affects the optimal labor supply and why. What is the elasticity of labor supply with respect to wage?

第三大題 (25 分) Solow Growth Model

In a closed economy, the basic Solow growth model consists of three key equations:

$$\begin{aligned} Y_t &= AK_t^\alpha L_t^{1-\alpha}; \\ K_{t+1} &= (1-\delta)K_t + I_t; \\ S_t &= sY_t; \end{aligned}$$

where Y_t , K_t , L_t , I_t , and S_t represent aggregate output, the stock of physical capital, aggregate labor, total investment, and aggregate saving in Country X at period t , respectively.

- A. (5 points) Use the basic Solow model to graphically illustrate Country X's steady state equilibrium.
- B. (5 points) "A steady state equilibrium is identical to a market equilibrium." True or false? Explain your answer.
- C. (5 points) Show that, Country X's capital-output ratio at steady state is constant over time.
- D. (10 points) The winners of 2024 Nobel Prize in Economics (Daron Acemoglu, Simon Johnson, and James Robinson) have demonstrated the importance of societal institutions for a country's prosperity. Suppose initially Country X was in a steady state equilibrium. At the beginning of 2025, Country X implemented a policy reform by introducing some good rules of law and institutions. According to the finding of the 2024 Nobel Prize winners, Country X will gain from the policy reform and enjoy more aggregate output. Please graphically illustrate how the policy reform will affect Country X. Specifically, plot Country X's new steady state equilibrium in your figure.

第四大題 (25 分) Two-period Model

A growing number of young people prefer having pets to having children. However, pets can accompany children as they grow up. The key to having pets or children lies in whether pets and children are substitutes or complements. Let's consider a representative agent of Country T in a two-period model. Assume that Country T is a small open economy. Thus, the wage rate (w) and interest rate (r) in Country T are taken as given. The representative agent cares about her consumption in period 1 (c_1), consumption in period 2 (c_2), the number of children she has (n), and the number of pets she raises (m). Specifically, her life-time utility is given by:

$$U = \log c_1 + \delta(n^\alpha m^{1-\alpha}) + \beta \log c_2$$

where $\delta > 0$ is the preference parameter of children and pets relative to consumption; and β is the subjective discount factor, $0 < \beta < 1$. Her preferences on the number of children and pets are nested by a Cobb-Douglas function with the parameter α . The agent has one unit of time in each period. In the first period, she works and allocates her labor income among raising children, having pets, her consumption in period 1, and her saving (s). The goods cost of raising a child is p , with a portion of it subsidized by the government. The portion is denoted as ρ . Similarly, the goods cost of having a pet is q , but the government imposes a proportional tax on the cost of raising a pet. Denote the tax rate as τ_m . To be simple, we assume the agent retires in the second period and only depends on her saving.

- A. (5 points) Write down the representative agent's maximization problem, including the budget constraints for the two periods.
- B. (5 points) Solve for the optimal pets-to-children ratio (m/n).
- C. (5 points) Solve for the optimal consumption for period 1.
- D. (10 points) In this setup, can an increase in the tax rate of pets boost fertility? Mathematically show your answer.