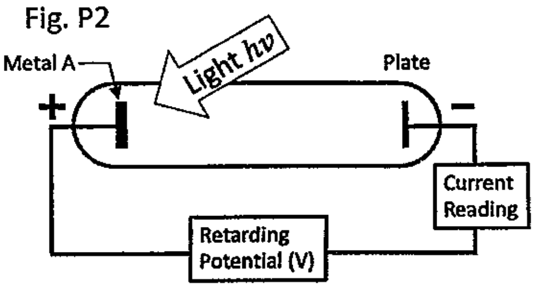
Planck's constant=6.626×10^{-34} J·s
Electron charge=1.602×10^{-19} C
Mass of an electron=9.1×10^{-31} kg
Mass of a proton=1.67×10^{-27} kg
Speed of light=3×10^{8} m/s
Vacuum permeability=1.257×10^{-6} N/A^2

1. Rocket A flies towards Earth at 0.9c, where c is the speed of light. The other one, called Rocket B, flies away from Earth at 0.5c. The proper length of the Rocket A is 200m and the proper length of the Rocket B is 300m. (a)(5pts) If the Earth is stationary, calculate the lengths of the Rocket A and Rocket B viewed from the Earth. (b)(8pts) What is the velocity of the Rocket B in the reference frame of the Rocket A? What is the length of the Rocket B viewed from the Rocket A?

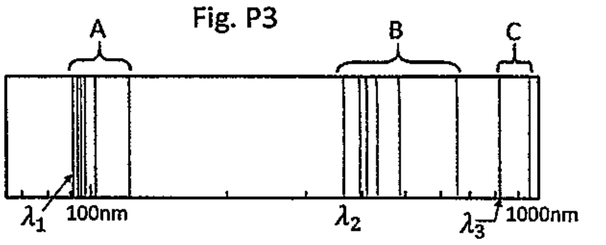
2. (8pts) An experiment was set up as shown in Fig. P2. A metal plate (made of metal A) is used as the anode, and a focused light is shined upon the plate. If we apply proper retarding potential (in volts) between the two plates in the vacuum tube, we will see the electrical current measured by the current reading equipment. The work function of the metal A is 2.3eV, and two different lights: case 1: blue (λ_1=420nm) and case 2: red (λ_2=630nm) impinge upon the metal A plate separately to test the currents. If the retarding voltage increases from zero to 10V, please sketch the current reading versus the retarding voltage for case 1 and case 2. In these plots, mark important information that you can obtain from the conditions in this question.

Fig. P2



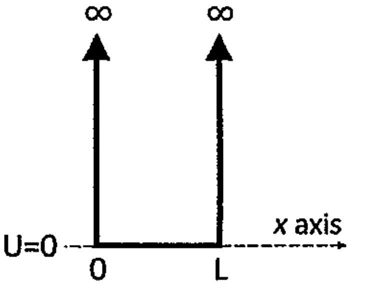
3. We obtained an atomic emission spectrum of hydrogen. Due to the limit of spectral resolution range, we only cover up to 1000nm of wavelength and its first three series in Fig. P3. (a)(3pts) $\lambda_1, \lambda_2, \lambda_3$ represent the shortest wavelengths of emission in the series A, B, and C, respectively. Please obtain their values using the Rydberg constant of $1.097 \times 10^{7} m^{-1}$. (b)(6pts) Now we know that different series of spectral lines correspond to the different final states of quantum transitions. Will the two adjacent spectral series have their emission wavelength range overlap to each other? (i.e. the shortest emission wavelength of one series is shorter than the other's longest emission wavelength). If this overlap does happen, provide an example between which two spectral series will this phenomenon happen (please specify their final state's quantum number)? If you think the emission spectrum of two adjacent spectral series won't overlap, please prove it.

Fig. P3



4. In a 1-D potential with infinite barriers and its width is L, as shown in the Fig. P4. (a) (3pts)First show the wavefunction for the n-th energy level in this structure via the time-independent Schrodinger equation. (b)(3pts) In this 1-D potential well, what is the expectation value of location variable x? (c) (9pts) Same situation as (b), what is the standard deviation of this variable x? The standard deviation is defined as: $\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ (d) (5pts) If L is reduced indefinitely to a very very small number(close to zero), will the standard deviation of x become negligible or even approach zero? Will this violate the uncertainty principle? (please state your reasons).

Fig. P4



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5. The emission spectrum from the Sodium (Na) atom consists of a closely spaced pair of spectral lines at wavelengths of 588.995 nm and 589.592 nm. This is due to the spin-orbit splitting of the sodium 3p level. Please draw an energy-level diagram and show the electronic transitions giving rise to these lines. In your diagram, label the atomic states that participate in this transition. Determine the magnitude of the spin-orbit energy based on this transition. (8pts)

6. A He–Ne laser is a type of gas laser whose high energetic gain medium consists of a mixture of helium and neon. Assume a He-Ne laser produces an output power of 3.5 mW with a diameter of 2.4 mm. The power can be expressed as $P = \frac{1}{\mu_0 c} E_0^2 A \sin^2(kz - \omega t)$, where A is the area of the laser spot size.

(a) Please calculate how many photons per second are emitted by the laser. (4pts)

(b) Please estimate the amplitude of the electrical field (E_0) of the light wave by the average power, i.e. $P_{av} = \frac{1}{T}\int_0^T P dt$. (4pts)

(c) If we have another incandescent light bulb that emits 100 W, please estimate the amplitude of the electric field at a distance of 1 m from the light bulb. (4pts)

7. The vibrational energy of CO is 0.2691 eV when the molecule contains the most common isotopes of carbon (mass = 12.00 u) and oxygen (mass = 16.00 u). The vibrational energy depends on the vibrational frequency f, which is determined by the bond's force constant k and the reduced mass μ of the molecule. Assume the vibrational energy of 0.2691 eV corresponds to the ground state (n=0).

(a) What would be the vibrational energy if the oxygen atom is replaced by its less abundant isotope with a mass of 18.00 u? (5pts)

(b) Calculate the vibrational frequency f if the carbon atom is replaced with radioactive carbon (C-14). (5pts)

8. Consider a block of metal at 0 K, where the electrons are spin-1/2 particles. The average energy of these electrons is E. Now imagine the electrons are replaced by particles with spin 1, while all other properties of the block of metal remain the same. Would the average energy of the spin-1 particles at 0 K be higher, lower, or the same as E? Explain your reasoning based on the principles of quantum mechanics and the distribution of particles in energy states. (8pts)

9. (a) In an intrinsic semiconductor, the Fermi energy at absolute zero temperature lies exactly in the middle of the energy gap between the valence band and the conduction band. When the temperature is increased to around room temperature, it is observed that there are Ne empty states in the valence band. At this temperature, is the number of filled states in the conduction band greater than, less than, or equal to Ne? Explain your reasoning. (4pts)

(b) If this intrinsic semiconductor is doped with donor atoms, the Fermi energy shifts closer to the conduction band. Suppose the temperature is raised to around room temperature for this doped semiconductor. Will the number of the number of filled states in the conduction band now be greater than, less than, or equal to Ne? Explain your reasoning. (4pts)

(c) How does the position of the Fermi energy relative to the conduction band influence the relative number of empty and filled states in the material at room temperature? (4pts)

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