國立臺灣大學 114 學年度碩士班招生考試試題

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

本試卷全部為多重選擇題，每題 5 分；每題答案可能不只一個，考生應作答於『答案卡』

1. Given two random variables, x and y with finite second moments, which of following statement(s) about independence is correct?
   (a) If x and y are independent with each other, then they are uncorrelated.
   (b) If x and y are uncorrelated with each other, then they are definitely independent of each other.
   (c) If $P(x=a \mid y=b)=P(y=b)$ then x and y are independent of each other.
   (d) If $E(x \mid y)$ is a constant, then x and y are independent of each other.
2. A random variable $x \sim N^{+}(0, \sigma^2)$, where N^{+} is a half-normal distribution that x is always positive and has a pdf, then we know:
   (a) $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right), -\infty < x < \infty.$
   (b) $f(x) = \frac{\sqrt{2}}{\sqrt{\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right), 0 < x < \infty.$
   (c) $E(x) = \sqrt{\frac{\sigma^2}{\pi}}.$
   (d) $Var(x) > \sigma^2.$
3. Which of following statement(s) about the Central Limit Theorem (CLT) is correct?
   (a) If $plim\,\bar{x} = \mu_x$, then CLT is held.
   (b) If N>30, then $\bar{x} \overset{A}{\sim} N(\mu_x, \sigma^2)$ under the conditions that the random variable x has finite mean and variance.
   (c) If N>30, then $\bar{x} \overset{A}{\sim} N(\mu_x, \frac{\sigma^2}{30})$ under the conditions that the random variable x has finite mean and variance.
   (d) $\bar{x}$ does not converge to normal distribution if x is a random walk process: $x_t = x_{t-1} + w_t, w_t \sim N(0,1)$.
4. Given cdf of a random variable x: $F_x(a) = \frac{a^2}{36}$, then we have…
   (a) The pdf $f_x(a) = \frac{a}{18}, 0 \le a \le 6.$
   (b) $E(x) = 4.$
   (c) $E(x^2) = 2$
   (d) $Var(x) = 2$
5. Let $u = (x-b)^2$, x is a random variable and $E[(x-b)^2]$ exists. Which of following statement(s) is correct?
   (a) $E(u)$ is minimal when b=0.
   (b) When b=0, u is the variance of x.
   (c) $E(u)$ is minimal when $b=E(x)$.
   (d) When $b=E(x)$, u is the variance of x.
6. A random sample $\{x_1, x_2, \ldots, x_N\}$ is sampled, where $x \overset{i.d.}{\sim} N(\mu_x, \sigma_i^2)$, which means x is independent distributed to a normal distribution (note that heteroskedasticity exists). A point estimator is calculated as $\tilde{x} = \sum_{i=1}^{N} a_i x_i;\ \sum_{i=1}^{N} a_i = 1$. Then which of following statement(s) is correct.
   (a) $\tilde{x}$ is unbiased to μ_x.

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(b) $\tilde{x}$ is a best linear unbiased estimator of μ_x.

(c) $\tilde{x}$ is a best unbiased estimator of μ_x.

(d) $\tilde{x}$ is a consistent estimator of μ_x, if a_i=1/N.

7. A random sample $\{x_1, x_2, \ldots, x_N\}$ is sampled, where $x \overset{i.i.d.}{\sim} N(\mu_x, \sigma^2)$. We define a downside standard deviation of x as $\hat{\sigma}_x^d = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \max(0, -x_i)^2}$, and $\hat{\sigma}_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_x)^2}$ is the classical standard deviation. Then we will obtain:
   (a) $\hat{\sigma}_x^d > \hat{\sigma}_x$.
   (b) $\hat{\sigma}_x^d < \hat{\sigma}_x$.
   (c) $\hat{\sigma}_x^d$ becomes larger when x is more positive skewed.
   (d) Both of $\hat{\sigma}_x^d$ and $\hat{\sigma}_x$ are biased estimators of population standard deviation σ.
8. A random variable $x \sim (\mu_x, 1)$, according to Chebyshev inequality, the lower bound of $P(\{|x - \mu_x| < 2\})$ is
   (a) 0
   (b) 0.75
   (c) 0.95
   (d) 0.99
9. Two random variables x and y have following relations: $y = b_0 + b_1 x + u$, and $x = a_0 + a_1 y + v$. Error terms u and v are independent of each other, which both obey standardized normal distribution. We can know…
   (a) OLS estimator $\hat{b}_1$ is a consistent estimator.
   (b) If $b_1 > 0$ and $a_1 > 0$, then OLS estimator $\hat{b}_1$ is downward inconsistent.
   (c) If $b_1 > 0$ and $a_1 < 0$, then OLS estimator $\hat{b}_1$ is downward inconsistent.
   (d) If $b_1 > 1$ and $a_1 > 0$, then OLS estimator $\hat{b}_1$ is upward inconsistent.
10. Using following OLS estimations (see table below) for regression model $Y_i = b_0 + b_1 X_i + b_2 D_i + b_3 X_i D_i + u_i$, in which X is a continuous variable, and D is a binary variable, please answer which of following statement(s) is correct?

Summary statistics

R-sq	0.90	Mean of Y	1
Adj. R-sq	0.89	Mean of X	0
N	50	Mean of D	0.6
		Mean of X*D	0.02

ANOVA

	DF	SS	MS	F
Regression	3	345.61	115.20	135.53
Residual	46	39.10	0.85	
Sum	49	384.71		

	Coeff	SD	t-stat
Intercept	1.14	0.21	5.51
X	1.80	0.27	6.74
D	-0.24	0.27	-0.91
X*D	1.44	0.32	4.53

(a) Average marginal effect for a unit increase in X is 1.8.

(b) Average marginal effect for a unit increase in X is 2.66.

(c) The mean squared error of $Y_i = b_0 + b_1 X_i + b_2 D_i + b_3 X D_i + u_i$ is smaller than a simple linear regression model: $Y_i = \beta_0 + \beta_1 X_i + v_i$

(d) $\widehat{Cov(X, D)} \cong 0.02$.

11. Let $\{(X_i, Y_i)'\}_{i=1}^q$ be a sequence of independently and $N(0, I_2)$-distributed random vectors. Define the random variable:

$$Z_i(q) = \frac{X_i}{\sqrt{\sum_{i=1}^q Y_i^2/q}}.$$

Which of the following is right?

(a) $\mathbb{E}[Z_i(q)] = 0$.

(b) $\mathbb{E}[Z_i^3(q)] = 0$.

(c) $\mathbb{E}[Z_i^4(q)] = 3$, as $q \to \infty$.

(d) $\mathbb{E}[Z_i^6(q)] = 15$, as $q \to \infty$.

12. Let $(Y_1, Y_2, \ldots, Y_n)'$ be a random vector with the distribution $N(0, \Sigma)$ and the covariance matrix:

$$\Sigma = \begin{bmatrix} 1 & \rho & \rho^2 & \ldots & \rho^{n-1} \\ \rho & 1 & \rho & \ldots & \rho^{n-2} \\ \rho^2 & \rho & 1 & \ldots & \rho^{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \ldots & 1 \end{bmatrix},$$

for some $\rho > 0$ and $n \geq 3$. Define the sample average:

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^n Y_i.$$

Which of the following is right?

(a) $\text{var}[\bar{Y}] = \frac{1}{n} + 2\sum_{i=1}^n (1 - \frac{i}{n})\rho^i$.

(b) $\lim_{n\to\infty} \text{var}[\bar{Y}] = 0$.

(c) $\text{var}[n^{1/2}\bar{Y}] < 1 + 2\sum_{i=1}^n \left(1 - \frac{i}{n}\right)\rho^i$.

(d) $\lim_{n\to\infty} \text{var}[n^{1/2}\bar{Y}] = 1$, if $\rho = n^{-1/2}$.

13. Let X be a $\chi^2(k)$-distributed random variable, and Y be a $N(0,1)$-distributed random variable. Suppose that X and Y are independent. Define the random variable:

$$W = X^{1/2}Y.$$

Which of the following is right?

(a) $\mathbb{E}[W^4] = 15$, if $k = 1$.

(b) $\mathbb{E}[W^4] = 30$, if $k = 2$.

(c) $\mathbb{E}[W^4] = 45$, if $k = 3$.

(d) None of the above choices (a)-(c).

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14. Let $\{(X_i, Z_i)'\}_{i=1}^n$ be a sequence of independently and $N(0, \Sigma)$-distributed random vectors, with the covariance matrix:

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 2 \end{bmatrix},$$

for some constant $\rho > 0$. Define the random variable:

$$Y_i = X_i^2 + Z_i,$$

for all i's. Consider a linear regression:

$$Y_i = \beta_0 + \beta_1 X_i + e_i,$$

where (β_0, β_1) is a parameter vector, and e_i is an error term, for all i's. Let $(\hat{\beta}_0, \hat{\beta}_1)$ be the ordinary least squares estimator of (β_0, β_1). Which of the following estimators is consistent for ρ, as $n \to \infty$?

(a) $\hat{\rho} = \hat{\beta}_1$.

(b) $\hat{\rho} = \hat{\beta}_1 + 29\left(\hat{\beta}_0 - 1\right)$.

(c) $\hat{\rho} = \left(\hat{\beta}_1 - \hat{\beta}_0\right)^2$.

(d) None of the above choices (a)-(c).

15. Let $\{X_i\}_{i=1}^n$ be a sequence of independently and $N(1, 2)$-distributed random variables. Define the sample average:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

Which of the following statistics has the limiting distribution $\chi^2(1)$, as $n \to \infty$?

(a) $\frac{n}{2}(\bar{X}^2 - 2\bar{X} + 1)$.

(b) $\frac{n}{8}(\bar{X}^4 - 2\bar{X}^2 + 1)$.

(c) $\frac{n}{16}(\bar{X}^6 - 2\bar{X}^3 + 1)$.

(d) $\frac{n}{32}(\bar{X}^8 - 2\bar{X}^4 + 1)$.

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16. Let $\{(Y_i, X_{1i}, X_{2i})'\}_{i=1}^n$ be a sequence of independently and $N(0, \Sigma)$-distributed random vector, where Σ is a 3×3 covariance matrix. Consider the following two regressions:

$$Y_i = \beta_0 + \beta_1 X_{1i} + e_{1i}$$

and

$$Y_i = b_1 X_{1i} + b_2 X_{2i} + e_{2i},$$

for all i's, with the parameters: β_0, β_1, b_1 and b_2 and the error terms: e_{1i} and e_{2i}. Let $(\hat{\beta}_0, \hat{\beta}_1)$ and $(\hat{b}_1, \hat{b}_2)$ be the ordinary least squares estimators of (β_0, β_1) and (b_1, b_2), respectively. Also, define the following two coefficients of determination:

$$R_1^2 = \frac{\sum_{i=1}^n (\hat{Y}_{1i} - \bar{\hat{Y}}_1)^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

and

$$R_2^2 = \frac{\sum_{i=1}^n (\hat{Y}_{2i} - \bar{\hat{Y}}_2)^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2},$$

where $\bar{Y} = n^{-1}\sum_{i=1}^n Y_i$, $\hat{Y}_{1i} = \hat{\beta}_0 + \hat{\beta}_1 X_{1i}$, $\hat{Y}_{2i} = \hat{b}_1 X_{1i} + \hat{b}_2 X_{2i}$, $\bar{\hat{Y}}_1 = n^{-1}\sum_{i=1}^n \hat{Y}_{1i}$ and $\bar{\hat{Y}}_2 = n^{-1}\sum_{i=1}^n \hat{Y}_{2i}$. Denote $\hat{e}_{1i} := Y_i - \hat{Y}_{1i}$ and $\hat{e}_{2i} := Y_i - \hat{Y}_{2i}$. Which of the following is right?

(a) $R_1^2 = 1 - \frac{\sum_{i=1}^n \hat{e}_{1i}^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$.

(b) $R_2^2 = 1 - \frac{\sum_{i=1}^n \hat{e}_{2i}^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$.

(c) $R_1^2 \neq 1 - \frac{\sum_{i=1}^n \hat{e}_{1i}^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$.

(d) $R_2^2 \neq 1 - \frac{\sum_{i=1}^n \hat{e}_{2i}^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$.

17. Let $(X, Y, Z)'$ be a random vector with the distribution $N(0, I_3)$. According to the Cauchy-Schwarz inequality, which of the following results is right?

(a) $\mathbb{E}|XY| \leq 1$.

(b) $\mathbb{E}|XY^2| \leq \sqrt{3}$.

(c) $\mathbb{E}|X^2Z^2| \leq 3$.

(d) $\mathbb{E}|X^2Y^3Z^3| \leq 15\sqrt{3}$

18. Let $\{(W_i, X_i, Y_i, Z_i)'\}_{i=1}^n$ be a sequence of random vectors that satisfies the following properties:

$$W_i = X_i^2 + Y_i^2 + Z_i^2,$$

$$\left[\begin{array}{c} Y_i \\ Z_i \end{array}\right] \Bigg| X_i \sim N\left(\left[\begin{array}{c} X_i \\ X_i^2 \end{array}\right], \left[\begin{array}{cc} X_i^2 & X_i^3 \\ X_i^3 & X_i^4 \end{array}\right]\right)$$

and X_i is $N(0,1)$-distributed, for all i's. Consider a linear regression:

$$W_i = \alpha_0 + \alpha_1 X_i + \alpha_2 X_i^2 + \alpha_3 X_i^3 + \alpha_4 X_i^4 + e_i,$$

where $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$ is a parameter vector, and e_i is an error term, for all i's Also, let $(\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4)$ be the ordinary least squares (OLS) estimator of $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$. Which of the following is the probability limit of $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$, as $n \to \infty$?

(a) $(0, 0, 2, 0, 4)$.

(b) $(0, 2, 0, 3, 4)$.

(c) $(0, 0, 3, 0, 2)$.

(d) None of the above choices (a)-(c).

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19. Let $\{X_i\}_{i=1}^n$ be a sequence of independently and $U(0,1)$-distributed random variables. Define the statistic:

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(X_i \leq x),$$

for some fixed $x \in (0,1)$, where $\mathbf{1}(A)$ is the indicator function which equals one if A is true (otherwise, zero). Which of the following is right?

(a) The limiting distribution of $n^{1/2}(F_n(x)/x - 1)$ is $N(0, 1/x - 1)$, as $n \to \infty$.

(b) The probability limit of $2F_n(x) - 1$ is $2x - 1$, as $n \to \infty$.

(c) The limiting variance of $n^{1/2}(F_n(x)/x^2 - 1/x)$ is $1/x^3 - 1/x^2$, as $n \to \infty$.

(d) The probability limit of $F_n(x)(1 - F_n(x))$ is the same as the limiting variance of $n^{1/2}(F_n(x) - x)$, as $n \to \infty$.

20. Let $\{(X_i, e_i)'\}_{i=1}^n$ be a sequence of independently and identically distributed random vectors with the properties: $X_i \sim N(0,1)$ and

$$e_i | X_i \sim N(0, X_i^2).$$

Define the random variable:

$$Y_i = \beta X_i + e_i,$$

where β is a constant, for all i's. Which of the following is right?

(a) The statistic $\frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2}$ is consistent for β, as $n \to \infty$.

(b) The statistic $\frac{\sum_{i=1}^n X_i^2 Y_i}{\sum_{i=1}^n X_i^3}$ is inconsistent for β, as $n \to \infty$.

(c) The statistic $\frac{\sum_{i=1}^n |X_i| Y_i}{\sum_{i=1}^n |X_i| X_i}$ is inconsistent for β, as $n \to \infty$.

(d) The statistic $\frac{\sum_{i=1}^n X_i Y_i |X_i|^{-2}}{\sum_{i=1}^n (X_i |X_i|^{-1})^2}$ is not less efficient than $\frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2}$, as $n \to \infty$.