國立臺灣大學 114 學年度碩士班招生考試試題

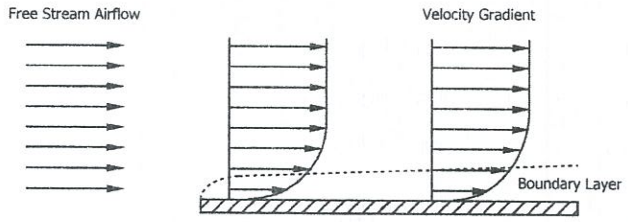
※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

Problem 1. (20%)

Briefly answer the following questions:

(a) Define the Reynolds number (*Re*) for a flow in a pipe (with a radius of *R*), and describe the *Re* criteria for laminar and turbulent flows, respectively. (5%)

(b) Write down the definition of the Prandtl number (*Pr*), and draw a scheme to illustrate the application of *Pr* in convective heat transfer. (5%) (Hint: refer to the following illustration of boundary layer theory.)



(website image)

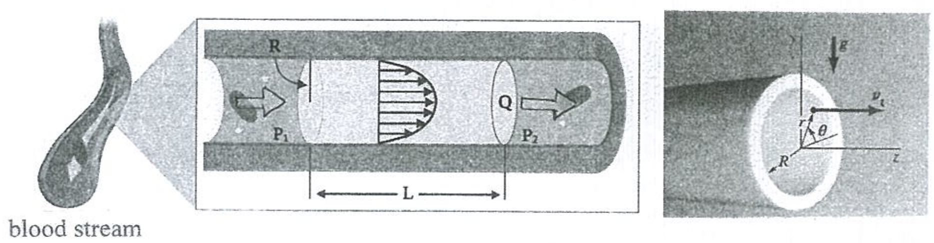
(c) Write down the definition of thermal diffusivity α, and compare its SI unit with that of a diffusion coefficient D for mass transfer. (5%)

(d) Derive Fick's 2nd law as given below from Fick's 1st law for a one-dimensional diffusion process. (5%)

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

Problem 2. (20%)

A healthy blood stream in a vessel is usually regarded as a laminar flow in a circular pipe as illustrated below, and the volume rate of flow Q in the blood vessel can be described by the Poiseuille equation.



(J. Cardiovasc. Dev. Dis. 2022, 9(9), 303.)

$$Q = \frac{\pi R^4 \Delta p}{8\mu L}$$

The Navier–Stokes equations in terms of cylindrical polar coordinates from a fluid mechanics textbook are also valid for the blood flow study, where the gravity terms can be neglected in such a study.

(r direction)

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \rho g_r + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right) - \frac{v_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

(θ direction)

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

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(z direction)

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

(a) If the mean velocity of the blood stream $\langle v_z \rangle$ and pressure drop $\Delta p/L$ in a simulated cardiovascular study are measured and correlated, what will be the slope of the $\langle v_z \rangle$ (in y-axis) *vs.* $\Delta p/L$ (in x-axis) plot? (5%)

(b) Prove that the velocity profile of v_z is as follows. (7%)

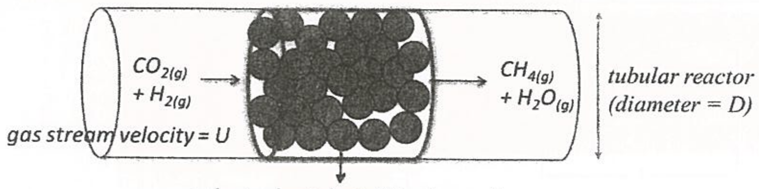
$$v_z=\frac{R^2}{4\mu}\left(\frac{\Delta p}{L}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$$

(c) Determine the shear stress τ_w on the blood vessel wall. (5%)

(d) Explain the effects of blood vessel narrowing (decrease in R) on the flow status (*e.g.*, more laminar or more turbulent) and τ_w (*e.g.*, larger or smaller). (3%)

Problem 3. (15%)

Consider a tubular plug-flow reactor (PFR with a diameter D) filled with specific spherical catalysts (with a diameter d) to convert $CO_{2(g)}$ to methane fuel by catalytic reduction for carbon capture and sustainable energy applications. The flow speed of the inlet gas stream is U, and the flow speed is uniform.



(a) Briefly draw (i) a typical f *vs.* log Re plot and (ii) a typical C_D *vs.* log Re plot in the Re range from very small to very large for the smooth surface case. (6%)

(b) Comment how fluid flow velocity U affects the friction loss caused by the reactor tube. (3%)

(c) Comment how the catalyst roughness affects the energy loss in the reactor operated at $Re = 10^4$ and $Re = 10^5$, respectively. Here Re is calculated based on d. (3%)

(d) Below is the Ergun equation. Briefly plot the pressure drop (in y-axis) *vs.* porosity (in x-axis) for this packed bed reactor. (3%)

$$\frac{\Delta p}{L}=\frac{150\bar{V}_0\mu}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}+\frac{1.75\rho\bar{V}_0^{\,2}}{\Phi_s\ D_p}\frac{1-\varepsilon}{\varepsilon^3}$$

Problem 4. (23%)

The heat sink technology based on enhanced heat transfer with finned surface (see below) has been widely applied to cool a variety of electronic devices, and it has re-gained much attention recently from GPU and AI industries to reduce power consumption and to facilitate high-performance computation.

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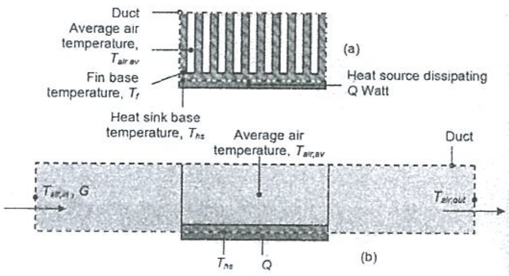
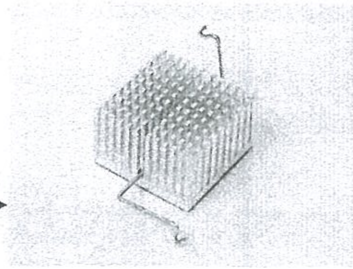
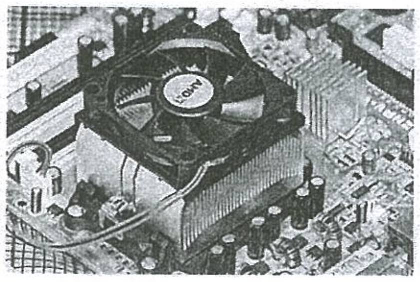






(from Wikipedia)

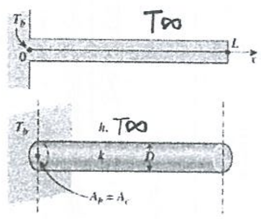
(a) The following "Fin equation" governs the heat trasnfer process in fins. Describe each term in the equation and how it is derived. (6%)

$$\frac{d^2T}{dx^2} - \frac{hp}{kA_c}(T - T_\infty) = 0$$

(b) In the case of "an infinitely long fin" ($T = T_\infty$), the temperature profile $T(x)$ is solved as below. Determine the steady rate of heat transfer from the entire fin by heat conduction. (6%)

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = exp(-mx)$$

$$\mathrm{m} = \sqrt{\frac{hp}{kA_c}}$$



(c) Prove that the fin efficiency η is $1/mL$ for the case of an infinitely long fin. The fin efficiency η is defined as the ratio of "actual heat transfer rate from the fin" to "ideal heat transfer rate from the fin if the entire film were at the base temperature T_b." (5%)

(d) It is known that the Nusselt number (Nu) can be correlated to the Reynolds (Re) and Prandtl (Pr) numbers by power laws like below, where a, b, and c are empirical constants.

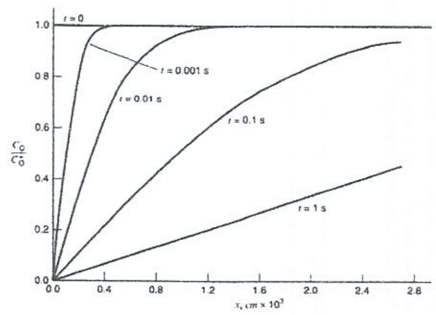
$$Nu = aRe^bPr^c$$

Comment how to design a high-efficiency heat sink from the perspectives of Nu. (6%)

Problem 5. (22%)

(a) An electrochemical process O + ne⁻ → R occurs at a planar inert electrode under diffusion control. Assume that there is no R present initally. The bulk concentraion and diffusion coefficient for species O are C_O^* and D_O, respectively. The concentration profile for species O, $C_O(x,t)$, is solved by Fick's 2nd law and plotted as follows, where x is the location in the electrolyte distant from the electrode.

$$C_O(x,t) = C_O^* erf\left[\frac{x}{2\sqrt{D_O t}}\right]$$



($x = 0$ is the electrode/eletrolyte interface, $D_O = 1 \times 10^{-5}$ cm²/s for this plot)

What are the initial condition and boundary conditions for Fick's 2nd law to solve such a profile? (5%)

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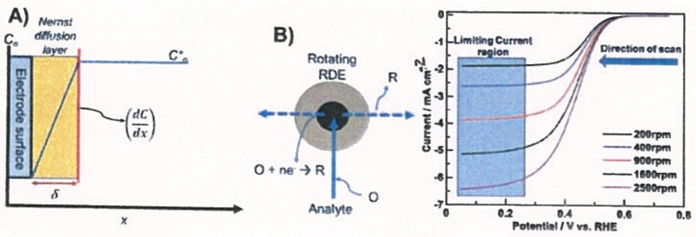
(b) The current response under diffusion control (at a suffiently negative potential) can be determined by correlating Faraday's electrolysis law and molar flux at the electrode/electrolyte interface ($x = 0$) as below.

$$-J_O(0,t) = \frac{i(t)}{nFA} = D_O\left[\frac{\partial C_O(x,t)}{\partial x}\right]_{x=0}$$

Determine $i(t)$. (6%)

(c) To obtain steady-state current responses, the rotating disk electrode (RDE) technique, as illustrated below, can be applied. The current responses for O + ne⁻ → R in a RDE system is perfectly modeled by the Levich equation, where i_L means a limiting current that is independent of applied potential.



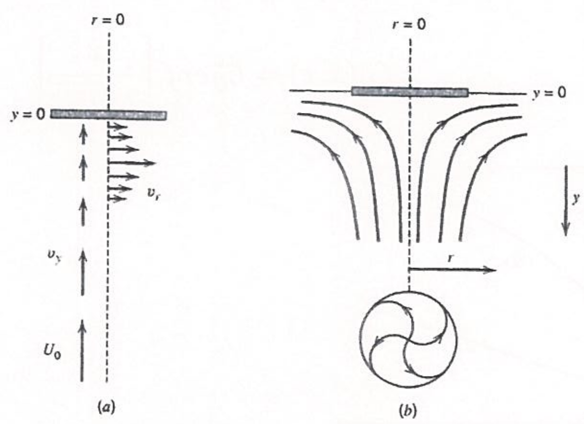
(https://doi.org/10.1515/cti-2021-0013)

$$i_L = 0.62nFAD_O^{2/3}\omega^{1/2}\nu^{-1/6}C_O^*$$

This implies that the convective diffusion transport of O to the electrode surface can be modeled by the film theory, and the apparent diffusion layer thickness becomes a constant. Determine (i) the diffusion layer thickness and (ii) mass transfer coefficient, respectively. (6%) (Hint: n, F, A, ω, ν are electron-transfer number, Faraday's constant, disk electrode area, angular velocity, kinematic viscosity, respectively.)

(d) The Levich equation is solved by a rather complicated PDE with steady-state approximation as described below. Explain the physical meaning of the following PDE. (5%)

$$v_r\left(\frac{\partial C_O}{\partial r}\right) + \frac{v_\phi}{r}\left(\frac{\partial C_O}{\partial \phi}\right) + v_y\left(\frac{\partial C_O}{\partial y}\right) = D_O\left[\frac{\partial^2 C_O}{\partial y^2} + \frac{\partial^2 C_O}{\partial r^2} + \frac{1}{r}\frac{\partial C_O}{\partial r} + \frac{1}{r^2}\left(\frac{\partial^2 C_O}{\partial \phi^2}\right)\right]$$



(a)Vector representation of flud velocities near a rotating disk. (b) Schematic resultant streamlines (or flows). (*from the Bard textbook*)

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