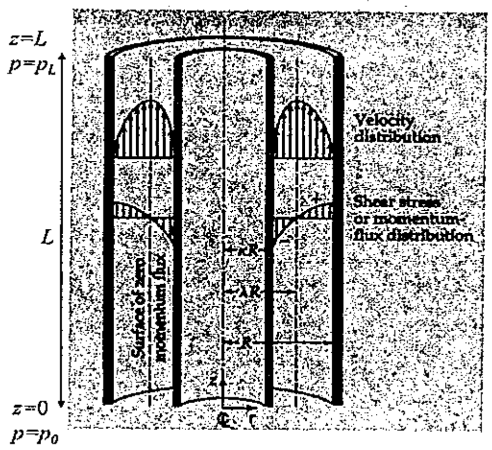
1. (25%)The steady-state axial flow of an incompressible liquid in an annular region between two coaxial cylinders of radii κR and R is shown below. The fluid is flowing upward in the tube – that is, in the direction opposed to gravity. The expression for the momentum-flux distribution is given

$$\tau_{rz} = \frac{(P_0 - P_L)R}{2L}\left[\left(\frac{r}{R}\right) - \frac{1-k^2}{2\ln(\frac{1}{k})}\left(\frac{R}{r}\right)\right]$$

where $P = p + \rho gz$.



(a) (5%) Derive the velocity distribution, $v_z(r)$.

(b) (5%) Determine the position r_{max} where the maximum velocity occurs.

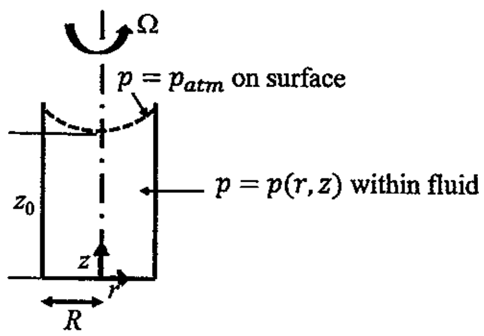
(c) (5%) Derive the average velocity, $\langle v_z \rangle$ and the mass rate of flow, ω.

(d) (5%) Derive the force exerted by the fluid on the solid surface, F_z.

(e) (5%) A horizontal annulus, 8.2 m in length, has an inner radius of 1.26 cm, and an outer radius of 2.8 cm. an aqueous solution with density 1286.45 kg/m^3 of and the viscosity of 0.05655 Pa·s. What is the volume flow rate when the impressed pressure difference is 3.7×10^4 Pa.

Note: $\int x\ln(x)dx = \left(\frac{x^2 \ln x}{2}\right) - \frac{x^2}{4} + C$

2. (25%) A liquid of constant density and viscosity is in a cylindrical container of radius R as shown below. The container is caused to rotate about its own axis at an angular velocity Ω. The cylinder axis is vertical. The system has reached steady state.



Cylindrical coordinates are appropriate for this problem, and the equations of motion are given

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

(a) (9%) Please simplify the equations of motion for different components (r, θ, z) by removing the unnecessary terms.

(b) (5%) Derive v_θ.

(c) (6%) Derive the pressure difference, $p - p_{atm}$.

(d) (5%) Derive the shape of the liquid-air interface, $z - z_0$.

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3. (15%) Consider a Couette viscometer consisting of two (vertical) concentric cylinders. The inner cylinder, of radius R, rotates at a constant angular velocity so that its linear velocity is U. The annular gap is filled with a viscous liquid. The outer cylinder, of radius κR ($\kappa > 1$), is held stationary. Assume that the gap distance is $(\kappa - 1)R$. The aim is to determine the temperature profile in the liquid resulting from viscous dissipation. Heat loss from the inner cylinder is negligible, whereas the liquid at κR loses heat to the ambient environment at T_a, with h as the heat transfer coefficient. You may neglect end effects and any variation with respect to the vertical coordinate z. Assume that $(\kappa - 1) \ll 1$; therefore, the curvature effect may be ignored. The system is at steady state, and the velocity profile can be taken to be linear with respect to the radius.

(a) (5%) Please write the expression for the flux of viscous heat dissipation.

(b) (5%) Under proper assumptions, please write the governing equation and the boundary conditions.

(c) (5%) Please solve for the temperature profile.

4. (15%) Consider a solid sphere of radius R of pure species A gradually dissolving into a stagnant fluid B at steady state. Its solubility is x_{A0} (in mole fraction); at positions far from the A sphere, the concentration is a constant value $x_{A\infty}$. Assume the fluid has constant density, constant mixture molarity, and constant diffusivity.

(a) (5%) Assuming that species A is slightly soluble, and the dissolution of the solid A does not lead to significant fluid flow, please solve for the concentration profile.

(b) (5%) Following (a), please define and calculate the Sherwood number.

(c) (5%) Suppose the assumptions in (a) do not hold, when a solid dissolves in a fluid, the solubility is so large that a mass transfer flux is generated on the solid surface, resulting in an average fluid velocity at the interface. This may lead to the inability to use the non-slip boundary condition for equation of motion when deriving the velocity profile of the system ($v_r|_{r=R} \neq 0$). Assuming mass average velocity is almost equal to molar average velocity $v_r \sim v_r^*$, what is the boundary condition of the velocity profile $v_r(r)$ at $r = R$?

5. (20%) A packed bed (with diameter D and length L) is filled with spherical solid packing of diameter d_p, where $D/d_p >> 1$. The porosity of the packed bed is ϕ. A pump delivers an aqueous solution with a volumetric flow rate Q at a slow flow velocity through this packed bed. The pressure drop across the system is Δp. Suppose the solute concentration in the solution is very low. Please answer the following questions:

(a) (4%) In a packed bed, the solute is affected not only by molecular diffusion, but also by factors such as non-uniform pathways between the packing particles and velocity distribution. All these effects cause the solute concentration distribution to spread out in the axial z-direction. If we use Fick's Law as a model to describe this phenomenon, the diffusion coefficient involved is called the axial dispersion coefficient E_D. All factors that contribute to the "broadening" of the solute in the axial direction are summarized in this coefficient. Please write down the partial differential equation (PDE) that describes the evolution of the solute concentration $C_i(t,z)$.

(b) (4%) Suppose the packing particles have a porous structure. Each particle has a porosity ε. Assume that the pore size is sufficiently small so that there is no flow convection occurring within the pores. Please explain how the PDE in part (a) should be modified. Let $\bar{C}_{i,pore}(t,z)$ be the average solute concentration in the particle pore structure at z.

(c) (4%) Suppose the solute has a diffusion coefficient D_p inside the pores of the particles, and its diffusion behavior can be described by Fick's Law. Making the appropriate assumptions, please write down the PDE that describes the evolution of the solute concentration $C_{i,pore}(t,r,z)$ inside the pores of a single particle at z.

(d) (4%) Please write down the mathematical relationship between $C_{i,pore}(t,r,z)$ and $\bar{C}_{i,pore}(t,z)$.

(e) (4%) Using the film model and a mass transfer coefficient k_f to describe mass transfer at the spherical surface of the particles, please write down the PDE relating $\bar{C}_{i,pore}(t,z)$ and $C_i(t,z)$.

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