1. Solve the following ordinary differential equations (ODEs)
   (a)(10%) $x^3y''' + x^2y'' - 2xy' + 2y = 3x^3\ln\ (x)$

   (b)(10%) $y'' - 5y' + 6y = 9\cos(3x)$

   (c)(10%) Find the general solution, $4y'' - 12y' + 9y = 2e^{1.5x}$
   then with the initial conditions, $y(0) = 1,\ \ y'(0) = 1$

2. (a) (5%) Find the Laplace transform, $\mathcal{L}\{e^{-t}[8\cosh(2t) - 3\sinh(4t)]\}$

   (b) (5%) Find the inverse Laplace transform, $\mathcal{F}(s) = \frac{2s-5}{s^2-6s+25}$

   (c) (10%) Solve the initial value problem using the Laplace transform.
   $$y'' - 2y' - 3y = u(t-1)$$
   $$y(0) = 0, \qquad y'(0) = -1$$
   Note: $u(t-1)$ is Unit Step Function (Heaviside Function)

   (d) (15%) Solve the heat equation using the Laplace transform.
   $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$
   Initial and boundary conditions:
   $$u(0,t) = 0, \qquad \lim_{x\to\infty} u(x,t) = 25, \qquad u(x,0) = 25$$

3. (15%) Find a power series solution using the Frobenius method and identify the first five terms of each of two linearly independent solutions:
   $$3xy'' + y' - y = 0$$

4. (10%) Verify whether the following function forms an orthogonal set. If it does, find its orthonormal set:
   $\{1, \cos(mx), \sin(mx)\},\ \ m = 1, 2, 3$ over the interval $[-\pi, \pi]$

5. (10%) Expand the following function in a Fourier series:
   $$f(x) = x^3;\ -\pi < x < \pi$$

見背面

Laplace Transforms of Selected Functions

$f(t)$	$\mathcal{F}(s)=\mathcal{L}\{f(t)\}$
a	$\frac{a}{s}$
t	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{s+a}$
$u(t-a)$	$\frac{e^{-as}}{s}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\mathrm{erfc}(\frac{a}{2\sqrt{t}})$	$\frac{1}{s}e^{-a\sqrt{s}}$

試題隨卷繳回