1. (10%) Find the general solution, $[2(y^3 - 2)]dx + 3xy^2dy = 0$, then with an initial condition, $y(3)=1$, find the unique solution.

2. (10%) Find the general solutions, $y^{(4)} + 2y'' + y = -\sin(x) + \cos(2x)$

3. Laplace transform,

(a)(5%) $L[t^2 cos(at)]$

(b)(5%) $L^{-1}[\frac{s}{(s+2)^2(s^2+2s+10)}]$

4. (10%) Find the general solution, $x^2y'' - xy' + y = 4x \ln(x)$

5. (10%) Find the solution of the system of differential equations,

$$\mathbf{X}'(t) = \begin{pmatrix} 3 & 0 & 1 \\ 9 & -1 & 2 \\ -9 & 4 & -1 \end{pmatrix} \mathbf{X}(t)$$

6. (10%) Given $z = -3 + 2i$
(a) Calculate the argument of z
(b) Express z in polar form

7. (15%) for $f(x) = 2x^2$ for $0 < x < 2\pi$, $f(x) = f(x + 2\pi)$
Express $f(x)$ using a Fourier series

8. (25%) For a 1D transient heat transfer system,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad for \quad 0 < x < l, \quad t > 0$$

$I.C.$ $u(x, t = 0) = 0$

$B.C.$ $\begin{cases} \text{adiabatic surface at } x = 0 \quad for\ t > 0 \\ u = 0 \text{ at } x = l \quad for\ t > 0 \end{cases}$

Find $u = u(x, t)$

The following integrals are for your information:

$$\int x^2 \cos(bx)\, dx = \frac{(b^2x^2 - 2)\sin(bx) + 2bx\cos(bx)}{b^3} + C$$

$$\int x^2 \sin(bx)\, dx = \frac{2bx\sin(bx) + (2 - b^2x^2)\cos(bx)}{b^3} + C$$

$$\int x \cos(bx)\, dx = \frac{bx\sin(bx) + \cos(bx)}{b^2} + C$$

試題隨卷繳回