

1. (20 points) Find the general solution to the differential equations

$$\begin{aligned}x_1'(t) &= x_2(t) + e^t, \\x_2'(t) &= -\frac{1}{4}x_1(t) + x_2(t).\end{aligned}$$

2. (20 points) Solve the integral equation

$$f(t) = 3t^2 - e^{-t} - \int_0^t f(s)e^{t-s} ds$$

for $f(t)$.

3. (20 points) Find the general solution of the differential equation

$$y''(t) - 5y'(t) + 4y(t) = 8e^t.$$

4. (20 points) Let $x(t)$ be the solution to

$$x'(t) = x(t)(0.5 - \sin x(t)), \quad x(0) = 0.01.$$

Find $\lim_{t \rightarrow \infty} x(t)$. Note that you have to give a proof to your answer.

5. (20 points) Find the power series solution of the initial value problem :

$$y'' - xy = 0, \quad y(0) = 3, \quad y'(0) = 0.$$

What is the radius of convergence of this power series ? Note that you have to write down the explicit formula for the coefficients.

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