(1) Suppose that $y[n] - 0.5y[n-1] = x[n]$.

(a) (5%) Please determine the causal sequence $h[n]$ such that $y[n] = x[n] * h[n]$ where * means the convolution.

(b) (5%) Also suppose that $3f[n] - f[n-1] = y[n] - y[n-1]$. Determine the causal sequence $g[n]$ such that $f[n] = x[n] * g[n]$.

(2) Determine the following convolution results (* means the convolution).

(a) (6%) $[3, 3, 2, 2, 1, 1] * [-1, 2, -1]$;

(b) (6%) $\text{sinc}(t) * \text{sinc}(3t) * \text{sinc}(5t) * \delta(t-6)$;

(c) (6%) $\text{sinc}^2(3t/2) * (1+\sin(2\pi t) + \cos(4\pi t))$.

(3) (12%) Determine $x[n]$ and $y[n]$ if their Z transform are as follows. Only consider the case where $x[n]$ and $y[n]$ are causal.

(a) $X(z) = \cos(z^3)$;　　(b) $Y(z) = \dfrac{z^{-3}}{12 - z^{-1} - z^{-2}}$.

(4) (10%) Suppose that $X(j\omega)$ is the continuous-time Fourier transform of $x(t)$ and $X(j\omega) \neq 0$ for $|\omega| < 6000\pi$, $X(j\omega) = 0$ for $|\omega| > 6000\pi$. Determine what is the lower bound of the sampling interval if we want to sample $y(t)$ and $z(t)$ as follows without the aliasing effect. In (a) and (b), * means the convolution and ′ means the derivative.

(a) $y(t) = x'(t) * \cos(4000\pi t)$　　(b) $z(t) = x(t-1)x(2t+2) * x(4t)$

In (5), (6), and (7), please simplify your answers as much as possible and write down the detailed steps

(5) Let $X(t) = r\sin(\alpha t + \Theta)$, where r and α are non-zero constants and Θ is uniformly distributed between 0 and 2π

(a) (4%) please find function $R(t, s) = \mathbb{E}[X(t)X(s)]$.　　(b) (5%) Assume that $R(0,3)=2$, please find $R(0.5,3.5)$

(6) Consider a binary communication system where a random bit b is transmitted. Suppose the 2×1 transmitted signal is

$$\mathbf{s} = \begin{bmatrix} (-1)^b \cdot d_{min}/2 \\ 0 \end{bmatrix},$$

where $b = 0, 1$ and $d_{min} > 0$. The received signal is $\mathbf{r} = \mathbf{s} + \mathbf{n}$, where noise

$$\mathbf{n} = [n_1\ n_2]^{\mathrm{T}}$$

is a white Gaussian random vector that is independent of $\mathbf{s}$ and T is the vector transpose.

(a) (4%) Assume that n_i has mean μ_i and variance $\sigma^2, i = 1, 2$. Please find the mean of noise vector $\mathbf{n}$.

(b) (2%) What is the definition of the covariance matrix of a random vector?

(c) (4%) Please find the covariance matrix of $\mathbf{n}$ using your answer in (b)

(d) (10%) Suppose that received

$$\mathbf{r} = [r_1\ r_2]^{\mathrm{T}}$$

Show that given r_1, r_2 is irrelevant; that is, r_2 is independent of $\mathbf{s}$

(7) Let $Q(x)$ be the probability that a zero-mean unit-variance Gaussian random variable is no less than x.

(a) (5%) The complementary error function is defined by

$$\text{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x \exp(-z^2) dz$$

Please express $Q(x)$ using $\text{erfc}(x)$. In other words, show how to transform function $\text{erfc}(x)$ to $Q(x)$?

(b) (4%) Please prove your answer in (a)

(c) (12%) For $1 > p_0 > 0$, find the value of r which minimizes

$$p_0 Q(1-r) + (1-p_0) Q(1+r).$$

Please express your answer in p_0.

試題隨卷繳回