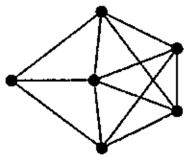
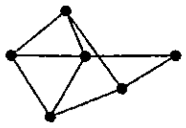
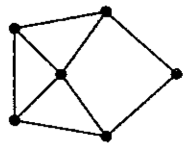
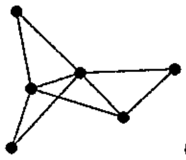
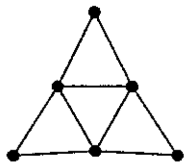
※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

1. (10%) ________ Which one of the following graphs has no Hamiltonian cycles?

(A)  (B)  (C)  (D)  (E) 

2. (10%) ________ Which solves $a_n = -a_{n-1} + 6\,a_{n-2}$ for a_n in terms of $a_0 = A$ and $a_1 = B$:

(A) $\frac{1}{5}[(-3)^n(2A-B)+2^n(3A+B)]$ (B) $\frac{1}{5}[(-3)^n(2A-B)+2^n(3A-B)]$ (C) $\frac{1}{5}[(-2)^n(3A-B)+3^n(2A+B)]$ (D) $\frac{1}{5}[(-2)^n(3A+B)+3^n(2A+B)]$ (E) $\frac{1}{5}[(-2)^n(3A-B)+3^n(2A-B)]$

3. (10%) ________ The generating function in partial fraction decomposition for the recurrence equation $a_n = a_{n-1} + 6\,a_{n-2}$ for a_n in terms of $a_0 = A$ and $a_1 = B$ is:

(A) $\frac{1}{5}\left[\frac{2A+B}{1-3x}+\frac{3A-B}{1+2x}\right]$ (B) $\frac{1}{5}\left[\frac{2A+B}{1-3x}+\frac{3A+B}{1+2x}\right]$ (C) $\frac{1}{5}\left[\frac{2A-B}{1-3x}+\frac{3A-B}{1+2x}\right]$ (D) $\frac{1}{5}\left[\frac{3A-B}{1-2x}+\frac{2A-B}{1+3x}\right]$ (E) $\frac{1}{5}\left[\frac{3A+B}{1-2x}+\frac{2A-B}{1+3x}\right]$

4. (10%) ________ The number of non-negative integer solutions of $x_1 + x_2 + \cdots + x_4 \le 7$ equals (A) 210 (B) 330 (C) 35 (D) 7 (E) 120

5. (10%) ________ If $|A| = 2^4$ and $|B| = 2^3$, how many functions from A to B are there? (A) 2^7 (B) 2^{12} (C) 2^{32} (D) 2^{48} (E) $2^{3^{16}}$

For problems 6-11, each problem may have multiple answers. Credits will be given only if all the answers are selected correctly.

6. (10%) Given $A = \begin{bmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix}$ and $x = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. If $\lim_{n\to\infty} A^n x = \begin{bmatrix} a \\ b \end{bmatrix}$, what is $2a+b$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. (10%) Given a linear transformation $T: P_2(R) \to P_2(R)$ defined by

$$T(f(x)) = f''(x) + 2f'(x) - f(x),$$

where $P_2(R)$ represents the real-valued 2nd-order polynomials. Assume M is the matrix representation of T with respect to the ordered basis. Please find the determinant of $(M^{-1}+2I)$, where I is the identity matrix with the same dimension as M. (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. (10%) Assume that A, B, and C are $n \times n$ matrices.

(A) trace((A+B)C) = trace(AC) + trace(BC)

(B) trace(AB) = trace(A)trace(B)

(C) trace(AB)=trace(BA)

(D) Given a 2×2 matrix, $\begin{bmatrix} a & b+c \\ b-c & -a \end{bmatrix}$ with trace zero has real eigenvalues if $a^2 + b^2 \ge c^2$.

(E) Suppose a 2×2 matrix other than the identity matrix satisfies $A^3 = I$. Then we have trace$(A) = 1$.

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9. (5%) Let S, D, and $Q \in M_n(R)$ represent a symmetric matrix, a diagonal matrix and an orthogonal matrix. Assume that their degree of freedom are d_S, d_D and d_Q, respectively. Find the value of $d_S - d_D - d_Q$.
(A) 0　(B) 1　(C) 2　(D) 3　(E) 4

10. (5%) Let $A \in M_{n\times n}$, which of the following statements are true?
(A) The nullspace of A is the same as the nullspace of A^TA.
(B) If A is orthogonal, $A + \frac{1}{2}I$ is invertible.
(C) If A has fewer than n distinct eigenvalues, then A is not diagonalizable.
(D) If $A = A^T$, then its eigenvalues are real and eigenvectors are orthogonal.
(E) If A is positive definite, we can always find a nonsingular U such that $A = U^2$.

11. (10%) Let T be a self-adjoint operator on a finite-dimensional inner product space V with distinct eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_k$. Let E_i be the eigenspace of T corresponding to λ_i, and let T_i be the orthogonal projection of V on E_i, $1 \le i \le k$.
(A) $\sum_{i=1}^{k} \lambda_i T_i = T$
(B) $V = E_1 \oplus E_2 \oplus \dots \oplus E_k$
(C) Given any polynomial g, $g(T) = g(\lambda_1)T_1 + g(\lambda_2)T_2 + \cdots + g(\lambda_k)T_k$.
(D) $\lambda_1, \lambda_2, \dots, \lambda_k$ are all real numbers.
(E) If $T(x) = Ax$, where $x \in \mathbb{C}^n$, then A is diagonalizable.

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