For the following problems, assume: $y' = \frac{dy}{dx}$ and $y' = \frac{d^2y}{dx^2}$

1.(10%)The following differential equation can be classified as:

$$xy' + xy + 2y = x^2 e^{-x}$$

A. linear homogenous
B. linear nonhomogenous
C. nonlinear homogenous
D. nonlinear nonhomogenous
E. cannot determine

2.(10%)For the following differential equation, $x = -3$ is which type of point?

$$(x^2 + x - 6)y' + (x + 3)y' + (x - 2)y = 0$$

A. ordinary
B. regular singular
C. irregular singular
D. none of these

3.(10%)Which is a solution of the following differential equation on interval (-5, 5)?

$$\frac{dy}{dx} = -\frac{x}{y}$$

A. $x^2 + y^2 = 25, -5 < x < 5$

B. $y = \sqrt{25 - x^2}, -5 < x < 5$

C. $y = -\sqrt{25 - x^2}, -5 < x < 5$

D. all of the above

E. none of these

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4.(10%)Which is a solution to the following differential equation?

$$y' + y = 0$$

A. $y = 0$

B. $y = e^{-x}$

C. $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$

D. both A and B

E. all of the above

5.(10%)Which is the general solution to the following differential equation?

$$y' - 3y' + 3y' - y = 0$$

Assume c_1, c_2, and c_3 are arbitrary constants.

A. $y = c_1 e^x + c_2 e^x + c_3 e^x$

B. $y = c_1 e^x + c_2 e^{x+1} + c_3 e^{x+2}$

C. $y = c_1 e^x + c_2 x e^x + c_3 x^2 e^x$

D. $y = c_1 e^x + c_2 \ln x \, e^x + c_3 (\ln x)^2 e^x$

E. none of these

6.(10%)Which is the general solution to the following differential equation?

$$x^2 y' - 2xy' - 4y = 0$$

Assume c_1 and c_2 are arbitrary constants.

A. $y = e^x \left[c_1 \cos(\sqrt{5}\, x) + c_2 \sin(\sqrt{5}\, x) \right]$

B. $y = c_1 e^{-x} + c_2 e^{4x}$

C. $y = x \left[c_1 \cos(\sqrt{5} \ln x) + c_2 \sin(\sqrt{5} \ln x) \right]$

D. $y = c_1 x^{-1} + c_2 x^4$

E. none of these

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7.(10%)Which is the general solution to the following differential equation?

$$2y' - 8y = 0$$

Assume c_1 and c_2 are arbitrary constants.

A. $y = c_1 \cos(2x) + c_2 \cos(2x)$

B. $y = c_1 \cosh(2x) + c_2 \cosh(2x)$

C. $y = c_1 \cos(2x) + c_2 \sin(2x)$

D. $y = c_1 \cosh(2x) + c_2 \sinh(2x)$

E. none of these

8.(10%)Which is the general solution to the following differential equation?

$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$$

Assume c_1, c_2, c_3, and c_4 are arbitrary constants.

A. $y = c_1 \cos(x) + c_2 \sin(x) + c_3 \cos(-x) + c_4 \sin(-x)$

B. $y = c_1 \cos(x) + c_2 \sin(x) + c_3 \, x \cos(x) + c_4 \, x \sin(x)$

C. $y = c_1 \cos(x) + c_2 \sin(x) + c_3 \, e^x \cos(x) + c_4 \, e^x \sin(x)$

D. $y = c_1 \cos(x) + c_2 \sin(x) + c_3 \, e^{-x} \cos(x) + c_4 \, e^{-x} \sin(x)$

E. none of these

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9.(10%)For the following differential equation on the interval $(0,\infty)$:

$$2xy'' + (1+x)y' + y = 0$$

which of the following equations are solutions?

A. $y = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n} x^n$

B. $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n\, n!} x^{(n+1/2)}$

C. $y = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{1\cdot3\cdot5\cdot7\cdots(2n-1)} x^n$

D. Both A and B

E. Both B and C

10.(10%)Which is the general solution to the following differential equation?

$$y'' + y = 4x + 10\sin(x)$$

Assume c_1 and c_2 are arbitrary constants.

A. $y = c_1\cos(x) + c_2\sin(x) + 4x$

B. $y = c_1\cos(x) + c_2\sin(x) + 4x - 5\sin(x)$

C. $y = c_1\cos(x) + c_2\sin(x) + 4x - 5\cos(x)$

D. $y = c_1\cos(x) + c_2\sin(x) + 4x - 5x\sin(x)$

E. $y = c_1\cos(x) + c_2\sin(x) + 4x - 5x\cos(x)$

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