1. Let W be the subspace of $\mathbf{R}^4$ spanned by the vectors $u_1 = (1, -2, 5, -3)$, $u_2 = (2, 3, 1, -4)$, $u_3 = (3, 8, -3, -5)$.

   (a) Find the basis and dimension of W. (8%)
   (b) Extend the basis of W to a basis of $\mathbf{R}^4$. (7%)

2. Show that $\begin{bmatrix} \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ \cos^2\alpha & \cos^2\beta & \cos^2\gamma \\ 1 & 1 & 1 \end{bmatrix}$ is not invertible for any values of α, β, and γ. (5%)

3. Suppose an operator matrix is $\begin{bmatrix} 51 & -12 & -21 \\ 60 & -40 & -28 \\ 57 & -68 & 1 \end{bmatrix}$. If 48 and 24 are eigenvalues of this matrix, find the third eigenvalue. (5%)

4. Find the volume $V(S)$ of the parallelepiped S in $\mathbf{R}^4$ determined by the following vectors. $u_1 = (1, -2, 5, -1)$, $u_2 = (2, 1, -2, 1)$, $u_3 = (3, 0, 1, -2)$, $u_4 = (1, -1, 4, -1)$. (5%)

5. Given $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$, find $\exp(A)$. (10%)

6. Let $A = \begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 4 \\ 2 & -3 & 5 \end{bmatrix}$. The characteristic polynomial of both matrices is $\Delta(t) = (t-2)(t-1)^2$.

   Find the minimal polynomial $m(t)$ of each matrix. (20%)

7. Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 7 & -5 \\ 2 & -5 & 8 \end{bmatrix}$. Find a non-singular matrix P such that $D = P^T AP$ is diagonal (10%), and the signature of A (10%).

8. The vectors $u_1 = (1,1,0)$, $u_2 = (1,2,3)$, $u_3 = (1,3,5)$ form a basis for Euclidean space $\mathbf{R}^3$. Find the matrix A that represents the inner product in $\mathbf{R}^3$ relative to this basis S. (10%)

9. Find an orthogonal matrix P whose first row is $u_1 = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$. (10%)

試題隨卷繳回