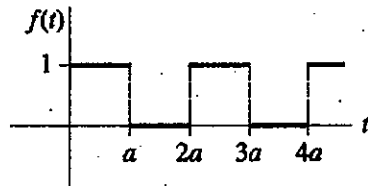


1. Solve the differential equation: $y'' + y' + y = \sec x$ (10%)
2. Find the general solution of $y' = 1 + (y - x)^2$ (10%)
3. Please find the zero-state response and zero-input response of the differential equation: $y'' + 4y = f(t), y'(0) = 0, y(0) = 1$, where $f(t)$ is shown as in following figure:

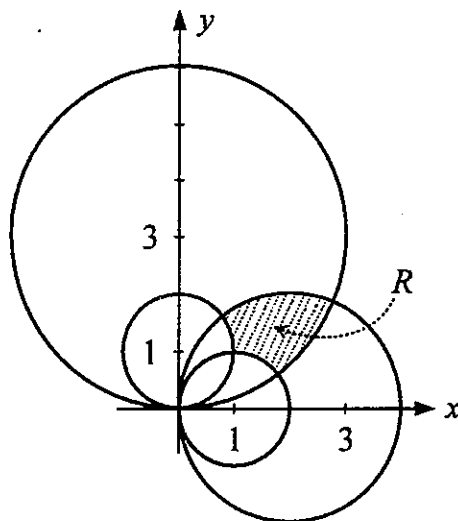


(15%)

4. Solve the system of linear differential equations $X' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} X$ (15%)

5. 試求雙重積分： $\iint_R \frac{1}{(x^2 + y^2)^3} dA = ?$ 此處， R 係由四個圓： $x^2 + y^2 = 2x$ ， $x^2 + y^2 = 4x$ ， $x^2 + y^2 = 2y$ ， $x^2 + y^2 = 6y$

所包圍而成的區域，如下圖所示。(15%)



6. 請利用固有函數展開法(Method of Eigenfunction Expansion)，求解下列的非齊次邊界值問題(Non-homogeneous Boundary-Value Problem)：

$$y'' + \frac{1}{4}y = \sin 2x, \quad y(0) = y(\pi) = 0. \quad (15\%)$$

7. 試求解下列的偏微分方程式：(20%)

$$k \frac{\partial^2 u}{\partial x^2} - h \frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0$$

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0$$

$$u(x, 0) = c_0, \quad 0 < x < 1$$

其中， k, h, c_0 均為常數。

試題隨卷繳回