國立臺灣大學 111 學年度碩士班招生考試試題

科目：機率統計

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1. (15 pts) Let $Y = \sum_{i=1}^{p} X_i$ with $X_1, \ldots, X_p$ being independent Poisson random variables with $E[X_i] = \lambda_i$, $i = 1, \ldots, p$. Find the distribution of Y.

2. (7 pts) (8 pts) Let the distribution of U given $T = t$ be $Uniform(0, t)$ and T follow an exponential distribution with rate $\lambda > 0$. Compute the mean and variance of U.

3. (15 pts) Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Compute the kth moment of $S_n^2 = \sum_{i=1}^{n}(X_i - \bar{X}_n)^2/(n-1)$, $k = 1, 2, \ldots$, where $\bar{X}_n = n^{-1} \sum_{i=1}^{n} X_i$.

4. Let $X_1, \ldots, X_n$ be a random sample from a Bernoulli distribution with $E[X_i] = \theta$, $i = 1, \ldots, n$.

(4a) (15 pts) Show that the maximum likelihood estimator of θ^2 is biased.

(4b) (15 pts) Find the uniformly minimum variance unbiased estimator.

5. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$, where σ^2 is a known positive constant.

(5a) (15 pts) Establish the uniformly most powerful size α test for the hypotheses $H_0 : \mu \leq \mu_0$ versus $H_A : \mu > \mu_0$.

(5b) (10 pts) Give a p-value for the above hypotheses based on the established test rule and the observed sample values $x_1, \ldots, x_n$.

試題隨卷繳回