國立臺灣大學111學年度碩士班招生考試試題

※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1. (20 points) Find the general solutions to the differential equation

$$x^2y'' + xy' + y = 0$$

for $x > 0$.

2. (20 points) Solve the differential equation

$$\begin{cases} y''(t) + y(t) = g(t), \\ y(0) = 0, \\ y'(0) = 1, \end{cases}$$

where

$$g(t) = \begin{cases} \dfrac{t}{2}, & 0 \le t < 6, \\ 3, & t \ge 6. \end{cases}$$

3. (20 points) Solve the initial value problem

$$\begin{cases} x'(t) = 5x(t) - y(t), \\ y'(t) = 3x(t) + y(t), \end{cases}$$

with initial the condition $(x(0), y(0)) = (1, 2)$.

4.(20 points) Find the solution of the initial value problem

$$4y'' - 4y' + y = 0,\ y(0) = 2,\ y'(0) = \frac{1}{3}.$$

5.(20 points) Solve the initial value problem

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)},\ y(0) = -1,$$

and determine the interval which the solution exists.

試題隨卷繳回