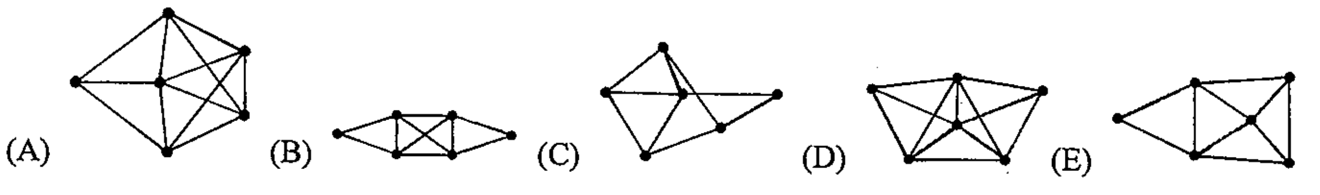
國立臺灣大學 111 學年度碩士班招生考試試題

1. (10%) ________ Which one of the following graphs has an Eulerian cycle?



2. (10%) ________ Which solves $a_n = a_{n-1} + 6\,a_{n-2}$ for a_n in terms of $a_0 = A$ and $a_1 = B$:

   (A) $\frac{1}{5}[(-3)^n(2A-B) + 2^n(3A+B)]$ (B) $\frac{1}{5}[(-3)^n(2A-B) + 2^n(3A-B)]$

   (C) $\frac{1}{5}[(-2)^n(3A-B) + 3^n(2A+B)]$ (D) $\frac{1}{5}[(-2)^n(3A+B) + 3^n(2A+B)]$

   (E) $\frac{1}{5}[(-2)^n(3A-B) + 3^n(2A-B)]$

3. (10%) ________ The generating function in partial fraction decomposition for the recurrence equation $a_n = -a_{n-1} + 6\,a_{n-2}$ for a_n in terms of $a_0 = A$ and $a_1 = B$ is:

   (A) $\frac{1}{5}\left[\frac{2A+B}{1-3x} + \frac{3A-B}{1+2x}\right]$ (B) $\frac{1}{5}\left[\frac{2A+B}{1-3x} + \frac{3A+B}{1+2x}\right]$ (C) $\frac{1}{5}\left[\frac{2A-B}{1-3x} + \frac{3A-B}{1+2x}\right]$ (D) $\frac{1}{5}\left[\frac{3A-B}{1-2x} + \frac{2A-B}{1+3x}\right]$ (E) $\frac{1}{5}\left[\frac{3A+B}{1-2x} + \frac{2A-B}{1+3x}\right]$

4. (10%) ________ The number of positive integer solutions of $x_1 + x_2 + \cdots + x_n = r$ equals (A) $\binom{r-1}{n-1}$ (B) $\binom{n+r-1}{n-1}$ (C) $\binom{r}{n}$ (D) r^n (E) $(rn)!$

5. (10%) ________ R is a non-symmetric relation on A if there exist $x, y \in A$ such that $(x,y) \in R$ but $(y,x) \notin R$. If $|A| = m$, how many non-symmetric relations on A are there? (A) $2^{(m^2-m)/2}$ (B) $2^{m^2} - 2^{(m^2+m)/2}$ (C) $3^{(m^2-m)/2}$ (D) $2^{(m^2+m)/2}$ (E) 2^{m^2-m}

6. (5%) ________ If A is similar to B, how many of the following statements are true?
   - A^{-1} is similar to B^{-1}.
   - A and B have the same eigenvalues.
   - A and B represent the same transformation with respect to different bases.
   - The nullity of A is the same as the nullity of B.

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. (5%) ________ How many of the following statements are true?
   - The inverse of an orthogonal matrix is orthogonal.
   - The product of two orthogonal matrices is an orthogonal matrix.
   - Every matrix with orthonormal columns is invertible.
   - If $A,\ B \in \mathbf{R}^{n\times n}$, and $A + iB$ is a unitary matrix, then A^TB is symmetric.

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. (5%) ________ Assume that $A \in \mathbf{R}^{n\times n}$ and $A^2 + 4A + 6I_n = O$. If $(A + 3I_n)^{-1} = aA + bI_n$. What is the value of $2a + b$? (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

見背面

國立臺灣大學 111 學年度碩士班招生考試試題

9. (5%) ________ Given a basis $S = \{(t-1)^3, (t-1)^2, (t-1), 1\}$ of a vector space $\mathbf{P}_3(t)$ of polynomials of degree≤ 3. If the coordinate of $x = 5t^3 - 4t^2 + 3t - 2$ with respect to S is (a, b, c, d), then $a + b + c + d =$?
(A) 26 (B) 27 (C) 28 (D) 29 (E) 30

10. (10%) ________ Given $A = \begin{bmatrix} 9 & -1 & 5 & 7 \\ 8 & 3 & 2 & -4 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & -1 & 8 \end{bmatrix}$. The largest eigenvalue is (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

11. (10%) ________ Given $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$. Assume that $B^3 = A$ and B has real eigenvalues. What is the matrix B?

(A) $\frac{1}{2}\begin{bmatrix} 1 + 4\sqrt[3]{5} & -1 + \sqrt[3]{5} \\ 3 + \sqrt[3]{5} & 1 - 2\sqrt[3]{5} \end{bmatrix}$ (B) $\frac{1}{3}\begin{bmatrix} 2 + \sqrt[3]{5} & 3 - \sqrt[3]{5} \\ -1 + 3\sqrt[3]{5} & 1 + 2\sqrt[3]{5} \end{bmatrix}$ (C) $\frac{1}{4}\begin{bmatrix} 3 + \sqrt[3]{5} & -1 + \sqrt[3]{5} \\ -3 + 3\sqrt[3]{5} & 1 + 3\sqrt[3]{5} \end{bmatrix}$

(D) $\frac{1}{4}\begin{bmatrix} 1 + 5\sqrt[3]{5} & 1 - \sqrt[3]{5} \\ 3 - 3\sqrt[3]{5} & 3 - \sqrt[3]{5} \end{bmatrix}$ (E) $\frac{1}{2}\begin{bmatrix} 1 + 5\sqrt[3]{5} & 3 - \sqrt[3]{5} \\ -3 + \sqrt[3]{5} & 1 - 3\sqrt[3]{5} \end{bmatrix}$

12. (10%) ________ Let $U = span\{(1,3,-2,2,3), (1,4,-3,4,2), (2,3,-1,-2,9), (2,4,-2,0,8)\}$ and $W = span\{(1,3,0,2,1), (1,5,-6,6,3), (2,5,3,2,1), (2,7,-3,6,3)\}$ be two subspaces of $\mathbf{R}^5$. The dimension of $U \cap W$ is (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

試題隨卷繳回