1. True/false questions. No need to explain.
   (a) Symmetric matrices are always diagonalizable. (2.5%).
   (b) Symmetric matrices are always invertible. (2.5%).
   (c) Eigenvectors of a symmetric matrix which come from different eigenspaces must be orthogonal. (2.5%).
   (d) Eigenvectors of a symmetric matrix which come from different eigenspaces must be linearly independent. (2.5%).

2. Let $A = \begin{bmatrix} 1 & 2 & 2 & 0 & 1 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Find the basis for the null space, row space, and column space of A. What is the rank of A? (10%).

3. Consider the linear system:

$$\begin{aligned} x_1 + x_3 &= q \\ x_2 + 2x_4 &= 0 \\ x_1 + 2x_3 + 3x_4 &= 0 \\ 2x_2 + 3x_3 + px_4 &= 3 \end{aligned}$$

in which p and q are parameters. Under what conditions (i.e. for what values of p and q) does this system have: (a) a unique solution? (b) no solution? (c) an infinite number of solutions? (10%).

4. Let A be a 3 × 3 matrix such that $Ax = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ has both solutions $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ as solutions. Find another solution to this equation. Please explain. (10%).

5. Prove that if λ is an eigenvalue of the matrix A, then λ^2 is an eigenvalue of the matrix A^2. (10%).

6. Let $A = \begin{bmatrix} -2 & 12 \\ -1 & 5 \end{bmatrix}$. Compute A^{10}. (10%).

7. Let $W \subset \mathbb{R}^4$ be the subspace of vectors (x_1, x_2, x_3, x_4) satisfying $2x_1 - x_3 + x_4 = 0$. Find an orthonormal basis for W. (15%).

8. Suppose you are doing a least squares regression in which you use an equation $y = C + D \cdot 2^x$ to fit the points $(x, y) = (0,6), (1,4), (2,0)$.
   (a) Find the coefficients C and D of the regression curve $y = C + D \cdot 2^x$. (10%).
   (b) What values should y be at times $x = 0, 1, 2$ so that the best curve is $y = 0$? (5%).

9. Suppose you are in a Pokémon game in which you can choose either Charizard or Pikachu to play the game. It is a multiplayer game. Assume x_k is the fraction of players who prefer Charizard to Pikachu at round k. The remaining fraction $y_k = 1 - x_k$ prefers Pikachu. At round $k + 1$, 1/5 of those who prefer Charizard change their mind. Also, at round $k + 1$, 1/10 of those who prefer Pikachu change their mind.
   (a) Create the matrix A to give $\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = A \begin{bmatrix} x_k \\ y_k \end{bmatrix}$. (5%).
   (b) Find the limit of $A^k \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as $k \to \infty$. (5%).

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