題號： 308 國立臺灣大學 111 學年度碩士班招生考試試題

科目： 單操與輸送

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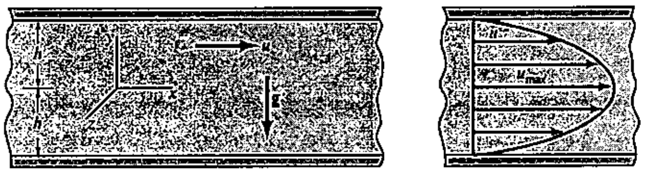
Problem 1. (20%)

Consider an incompressible Newtonian fluid flowing through a gap (height = 2 h) between two parallel plates. The fully developed laminar flow velocity profile u(y) can be determined by solving the x-component Navier-Stokes equation, where u, v, and w are fluid velocity components along x, y, and z direction, respectively.



$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

(a) Explain that the above equation is a result of momentum balance. (5%)

(b) Prove the velocity profile below. (5%)

$$u=\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2-h^2)$$

(c) Show that the volume rate of flow is proportional to h^3. (5%)

(d) Use a schematic to illustrate how the velocity profile u(y) changes for turbulent flow. (5%)

Problem 2. (20%)

(a) Explain fluid friction loss from the Bernoulli equation. (5%)

(b) Write down the physical significance and unit of the Fanning friction factor as defined below. (5%)

$$f\equiv\frac{\tau_w}{\rho\bar{V}^2/2}$$

(c) Justify the following equation for a laminar pipe flow, where Δp_s is the pressure drop due to skin friction, τ_w is the shear stress at the wall of the pipe, D is diameter of the pipe, and L is the length of the pipe. (5%)

$$\Delta p_s=4(\tau_w/D)\cdot L$$

(d) Use a schematic to compare how f varies with Re for laminar flow and turbulent flow, respectively. (5%)

Problem 3. (20%)

Consider that a thick beef steak is being baked on a hot plate with a surface temperature T_S. Before baking, the raw beef steak is placed at room temperature (T_O) for a sufficient long while, so the temperature distribution is uniform throughout the raw steak initially. Here we only model the first-stage of steak baking (*i.e.*, without flip-side), and the steak thickness is sufficiently large so that the following semi-infinite transient heat conduction model can be applied.

$$\frac{T-T_0}{T_S-T_0}=\mathrm{erfc}\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

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(a) Write down the physical meaning and SI unit of α in the above equation. (5%)

(b) Give the partial differential equation and related initial and boundary conditions for obtaining the above temperature profile model. (5%)

(c) Use a schematic to compare the temperature profiles $T(x)$ at small, middle, and large time t. (5%)

(d) Determine the heat flux required as a function of time $q(t)$ to bake the steak. (5%)

Problem 4. (20%)

(a) Derive the Fick's second law from the Fick's first law. (7%)

(b) Below is the Einstein diffusion equation.

$$c(x,t) = \frac{c_0}{\sqrt{4\pi Dt}} exp\left(\frac{-x^2}{4Dt}\right)$$

Show that the mean squared displacement of a molecule for one-dimensional diffusion is 2Dt. The following expoential integrals are useful. (7%)

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \qquad \int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

(c) For sucrose in water at 25 °C, $D = 5 \times 10^{-6}\ cm^2 \cdot s^{-1}$. Assume that in a region of an unstirred aqueous solution of sucrose the molar concentration gradient is $-0.2\ M \cdot cm^{-1}$. The amount of sucrose molecules passing through a 10 cm^2 window in 10 min is X μmol. X = ? (6%)

Problem 5. (20%)

Serum creatinine (denoted as A), a detectable byproduct of muscle metabolism, is viewed as a biomarker of kidney health because it is removed from the blood mainly by the kidneys (the so-called "creatinine clearance"). It is known that creatinine has a constant half-life ($t_{1/2}$, hr) while removed in the kidneys (*i.e.*, obeying first-order kinetics). Now, an organ chip as well as a microfluidic PFR (plug-flow reactor) made of 3D human kidney cell culture coated on the microfluidic channel walls is fabricated for creatinine metabolism study. The microfluidic PFR has a cross-section area of S (μm^2) and a length of L (cm), and an artificial blood stream containing a creatinine concentration of C_{A0} is pumped into the kidney-PFR chip at a constant volumetric flow rate v_0 (μl/hr) for the study.

(a) Draw a schematic to illustrate the PFR-mimic microfluidic kidney chip for creatinine removal study. (5%)

(b) Perform mass balance for a PFR reactor to relate the reactor volume to conversion X for a first-order chemical reaction. (5%)

(c) Show that the length of the microfluidic PFR kidney (L) can be modeled as follows. (5%)

$$L = -\left(\frac{t_{1/2}}{ln2}\right)\left(\frac{v_0}{S}\right) ln(1-X)$$

(d) Comment what would be different if the microfluidic organ chip was modeled by a packed-bed reactor instead of a PFR. (5%)