1. Consider a system G: $\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \end{cases}$

(1) Derive the state response $x(t)$ to an initial state $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ when $u(t)=0$ 【計分：10分】

(2) Derive the transfer function G(s) of the system 【計分：10分】

(3) Sketch the Bode plot of G(s) 【計分：10分】

(4) Refer to **Fig. A** and utilize the Nyquist criterion to discuss the stability of the closed loop system if the controller is designed as D(s)=K 【計分：10分】

(5) If the controller in **Fig. A** is designed as $D(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$, please find the parameters of the second order controller so that the closed-loop poles are allocated at $s = -\sqrt{3} \pm j, -10, -10, -10$. 【計分：10分】

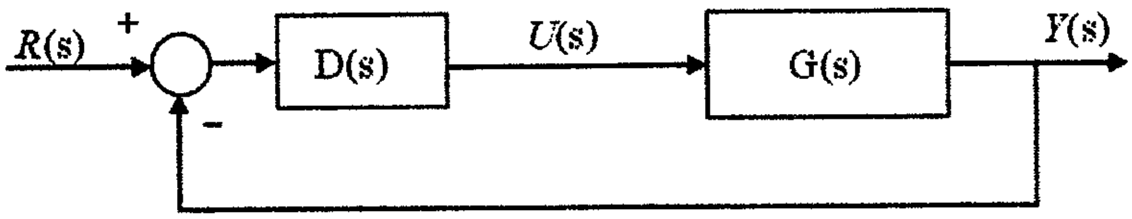

Fig. A

2. The block diagram of a feedback control system is shown in **Fig. B**. (1) Apply the gain formula of signal flow graph (SFG) directly to the block diagram to find the transfer functions: $\left.\frac{Y(s)}{R(s)}\right|_{N=0}$ and $\left.\frac{Y(s)}{N(s)}\right|_{R=0}$. 【計分：10分】 Express $Y(s)$ in terms of $R(s)$ and $N(s)$ when both inputs are applied simultaneously.【計分：2分】(2) Find the desired relation among the transfer functions $G_1(s)$, $G_2(s)$, $G_3(s)$, $G_4(s)$, $H_1(s)$, and $H_2(s)$ so that the output $Y(s)$ is not affected by the disturbance signal $N(s)$ at all.【計分：3分】

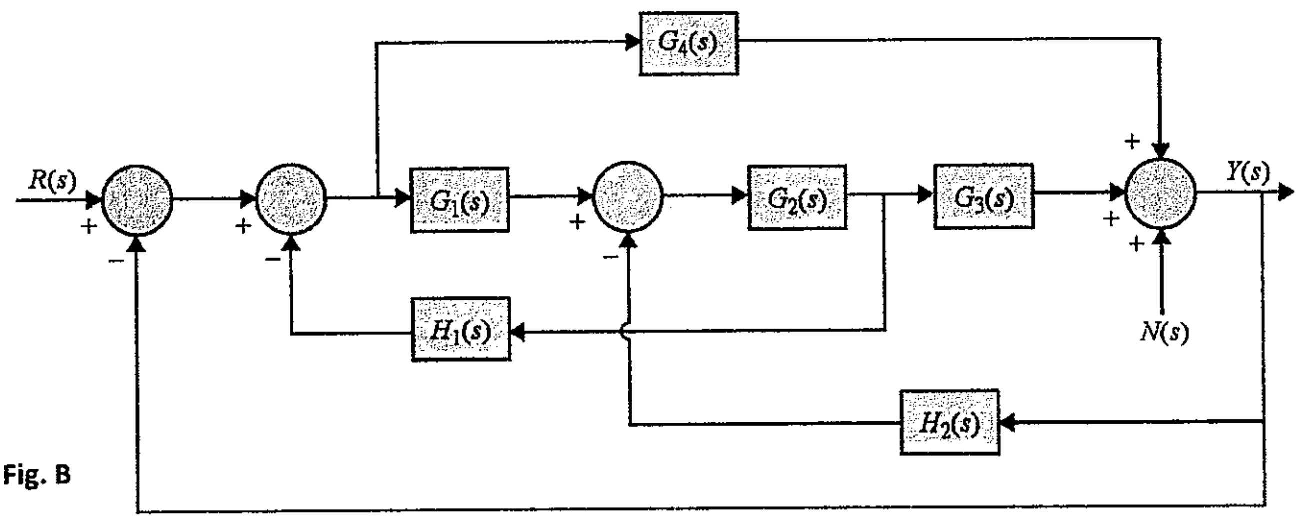

Fig. B

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3. Two pendulums, coupled by a spring, are to be controlled by two equal and opposite forces *u*, which are applied to the pendulum bobs, as shown in **Fig. C**. Assume that the displacement angles are small enough that the spring always remains horizontal. If the rods with the length of *l* are massless and the spring is attached to the rods *a* from the top. (1) Derive the equations of motion. 【計分：4分】(2) Set state variables as $x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$, and $x_4 = \dot{\theta}_2$, find the state equation of the system. 【計分：4分】 (3) Show that the system is uncontrollable. 【計分：8分】 Can you associate a physical meaning with the controllable and uncontrollable modes? 【計分：2分】 (4) Is there any way that the system can be made controllable? 【計分：2分】

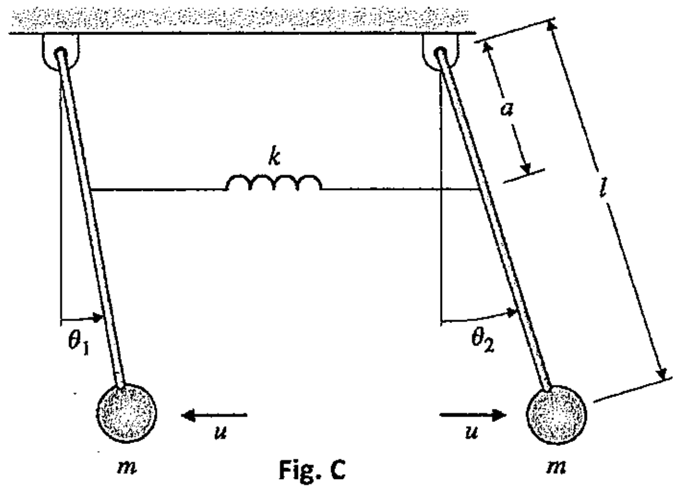

Fig. C

4. A controlled process is modeled by the following state equations:

$$\frac{dx_1(t)}{dt} = x_1(t) - 4x_2(t),\quad \frac{dx_2(t)}{dt} = 5x_1(t) + u(t), \text{ and } \quad y = x_1(t).$$

The control *u*(*t*) is obtained from state feedback such that $u(t) = -k_1x_1(t) - k_2x_2(t) + r(t)$, where k_1 and k_2 are real constants. (1) Determine the region in the k_1-versus-k_2 parameter plane in which the closed-loop system is asymptotically stable. 【計分：4 分】 (2) Find the loci in the k_1-versus-k_2 plane on which the overall system has a damping ratio of 0.707 and natural undamped frequency equals to 8 rad/sec.【計分：4 分】(3) Find the values of k_1 and k_2 such that damping ratio equals to 0.707 and peak time of the unit-step response is π/3 sec. 【計分：2 分】(4) Let the error signal be defined as *e*(*t*) = *r*(*t*) – *y*(*t*). Find the steady-state error when *r* (*t*) = unit step input and k_1 and k_2 are at the values found in (3). 【計分：2 分】 (5) Find the locus in the k_1-versus-k_2 plane on which the steady-state error due to a unit step input is zero. 【計分：3 分】

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