1. (50 points) Intertemporal Optimal Consumption and Saving

A consumer lives for two periods. His current income is $Y_t = 100$, and his income next period is $Y_{t+1} = 121$. Suppose the real interest rate is 10%. Assume he has the log utility function. Calculation is up to two decimal points. You must show your deriving process.

$$U = \log C_t + \beta \log C_{t+1}.$$

a) (6 points) What is the intertemporal budget constraint equation and his lifetime income? Plot the intertemporal budget constraint, where the vertical axis is C_{t+1} and the horizontal axis is C_t, mark the intercepts and the slope clearly.

b) (3 points) Suppose $\beta = 1$, what is the Euler equation, optimal consumption C_t, C_{t+1}, and saving? Plot your solution, the optimal indifference curve, on the intertemporal budget constraint.

c) (8 points) Suppose the consumer faces a no-borrowing constraint. That is, he can only save. Under the no-borrowing constraint, what is the Euler equation, the optimal consumption C_t, C_{t+1}? Plot your solution on the new intertemporal budget constraint along with the no-borrowing constraint.

d) (8 points) Following from b), suppose the interest rate for saving and the deposit is still 10%. However, when one borrows, the borrowing interest rate is 50%. What is the Euler equation, the optimal consumption C_t, C_{t+1}? Plot your solution on the intertemporal budget constraint with different interest rates for the saving and borrowing.

Now assume the utility function has the following form

$$U = \min(C_t, C_{t+1})$$

The real interest rate is 10% and $\beta = 1$. With this utility function, utility equals the minimum of period t and t+1 consumption. For example, if $C_t = 10$ and $C_{t+1} = 15$, the $U = 10$. This means C_t and C_{t+1} are perfect complements.

e) (2 points) For this perfect complement utility function, you cannot use calculus or Euler equation to characterize optimal behavior. Instead, think about it without math. What is the condition between C_t and C_{t+1} when the consumer is behaving optimally?

f) (3 points) Use the condition you have in e) to solve for the optimal consumption C_t and C_{t+1}. Is he a saver or a borrower?

g) (10 points) What is the value of marginal propensity to consume (MPC) at t?

h) (10 points) Derive the relation between the optimal C_t and the interest rate. Is the current optimal consumption decreasing in the real interest rate?

2. (16 points) True or False? Explain your answer. You will not obtain any credits unless you provide explanations to your answer.

a. (4 points) In the Solow model with exogenous labor-augmenting technological progress (at the growth rate g), output per worker still grows at the rate g. Therefore, we conclude that no steady state exists in such economy.

b. (4 points) The Solow model predicts that countries with lower population growth rates will have lower levels of income per worker in the long run because of the lack of labor forces.

c. (4 points) In Solow model, saving leads to higher output per worker; therefore, an economy can enjoy

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persistent growth forever by having a persistent increase in its saving rate.

d. (4 points) In the money market, money demand and money supply are brought into equilibrium by the real interest rate adjustment.

3. **(12 points) The basic Solow model without growth**

Suppose Country A's initial steady-state capital stock is k^*, which is larger than its Golden Rule capital stock (k^*_{gold}). The policymakers in Country A introduce a policy to adjust the saving rate (s) at time t_0 so that the adjustment leads to the Golden Rule steady state. What happen to consumption per person (c), output per person (y), and investment per person (i) during the transition from the initial steady state to the Golden Rule steady state? Draw a transition path for the variables c, y, and i, respectively. Please specify the time point t_0 in your graphs.

4. **(22 points) Two-period Model with Endogenous Fertility**

Consider a representative consumer in the two-period model. Her life-time utility is given by:

$$U=log(c_1)+log(l_1)+log(n)+\beta log(c_2),$$

where β is the subjective discount factor and $0<\beta<1$. She derives utility from her own consumption in both periods (c_1 and c_2). Besides, we assume that the consumer works in the first period but is retired in the second period. Therefore, in the first period, she allocates her one unit of time between leisure (l_1), raising children, and working. The wage rate of working is w. No utility from leisure in the second period. As she does not work in the second period, she consumes her own saving (s). The consumer also chooses the number of children she wants (n) in the first period and obtain utility from having children. However, children are costly. There are two types of child-rearing cost, both happening in the first period only. The first one is the food cost (per child), $p>0$. The second one is the time cost of raising children (per child). That is, the consumer has to spend the fraction ($\psi>0$) of her time on raising a child. Finally, the interest rate is denoted as r.

a. (4 points) Write down the representative consumer's maximization problem, including the budget constraints for the two periods.

b. (2 points) Write down the life-time budget constraint for the representative consumer.

c. (10 points) Solve for the optimal decisions on saving (s) and the number of children (n).

d. (6 points) Are children normal goods to the representative consumer? Why or why not? Mathematically show your answer.

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