1. Calculate the divergence of each vector functions:

(a) $\mathbf{F} = \mathbf{i}2yz + \mathbf{j}3xz + \mathbf{k}4xy$, (5%)

(b) $\mathbf{F} = \left(-\mathbf{i}xy + \mathbf{j}x^2\right)/\left(x^2 + y^2\right)$, $(x, y) \neq (0, 0)$ (5%)

2. Calculate the curl of each vector functions:

(a) $\mathbf{F} = \mathbf{i}z^2 + \mathbf{j}x^2 - \mathbf{k}y^2$, (5%)

(b) $\mathbf{F} = \left(\mathbf{i}x + \mathbf{j}y + \mathbf{k}z\right)/\left(x^2 + y^2 + z^2\right)^{3/2}$, $(x, y) \neq (0, 0)$ (5%)

3. Evaluate the line integral $\int_C \mathbf{F} \cdot \hat{\mathbf{t}}\, ds$, where $\mathbf{F} = (y^2 z)\mathbf{i} + (2xyz)\mathbf{j} - (2z + xy^2)\mathbf{k}$, curve C is defined parametrically by

$$\left.\begin{aligned} x &= \sin \pi t + t \\ y &= t(2t^3 - t^2) \\ z &= 2^t - 1 \end{aligned}\right\} \quad \text{for } 0 \leq t \leq 1$$

from the point (0,0,0) to the point (1,1,1), and $\hat{\mathbf{t}}$ is the unit tangent vector along curve C. You may use the Stokes' theorem to help you evaluate this line integral. (20%)

4. Find the general solution y(*t*) for the given differential equation,

$$\frac{dy}{dt} + \frac{2t}{1+t^2} y = \frac{1}{1+t^2}. \quad (10\ \%)$$

5. Find the solution y(*t*) for the given initial-value problem,

$$\cos y \frac{dy}{dt} = \frac{-t \sin y}{1+t^2}, \text{ and } \ y(1) = \frac{\pi}{2}. \quad (10\%)$$

6. Solve this ordinary differential equation, $\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 5y = 0$, under the initial conditions as $y(0) = 0, y'(0) = 2$. (20%)

7. Find the inverse matrix $\mathbf{A}^{-1}$, where $\mathbf{A}$ is defined as $\mathbf{A} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$. (20%)

試題隨卷繳回