In this exam:

- The height of a singleton tree is defined as 0.
- The order of a node in a tree is defined as the number of its children.
- The order of a tree is defined as the maximum order among its nodes.
- Unless otherwise specified, all logarithms are taken with respect to base 2, i.e., $\log_2$.

一、(68%)單選題（每題 4 分，答錯不倒扣）

Please choose the best possible answer in each of the following questions. In questions that ask for time complexities, please choose the tightest bound.

1. Consider a hash table with 13 slots. Suppose we use linear probing as the collision resolution strategy, where the i'th probe position (where i = 0, 1, 2, ...) for a key k is given by the function
$$h(k,i) = (k+i) \pmod{13}$$
Suppose we insert the following 9 keys into the hash table in the exact sequence: 66, 4, 84, 91, 100, 72, 70, 37, 61. What is the index of the slot storing the key 61?
(A) 2 (B) 8 (C) 9 (D) 10 (E) None of above
2. The worst case time complexity of constructing a binary heap from an unordered array of n items is
(A) O(n) (B) O(n log*n) (C) O(n log n) (D) $O(n^2)$ (E) $O(n^2 \log n)$
3. In a binary heap of size n, the worst case time complexity of one delete-min operation is
(A) O(1) (B) O(log*n) (C) O(log n) (D) O(n) (E) O(n log n)
4. The worst case time complexity of one union operation upon two binary heaps of total size n is
(A) O(1) (B) O(log*n) (C) O(log n) (D) O(n) (E) O(n log n)
5. In a binomial heap of size n, the worst case time complexity of n consecutive insert operations is
(A) O(n) (B) O(n log*n) (C) O(n log n) (D) $O(n^2)$ (E) $O(n^2 \log n)$
6. In a binomial heap of size n, the worst case time complexity of one insert operation is
(A) O(1) (B) O(log*n) (C) O(log n) (D) O(n) (E) O(n log n)
7. In a binomial heap of size n, the worst case time complexity of one find-min operation is
(A) O(1) (B) O(log*n) (C) O(log n) (D) O(n) (E) O(n log n)
8. In a Fibonacci heap of size n, the worst case time complexity of one insert operations is
(A) O(1) (B) O(log*n) (C) O(log n) (D) O(n) (E) O(n log n)
9. In a Fibonacci heap of size n, the worst case time complexity of one decrease-key operations is
(A) O(1) (B) O(log*n) (C) O(log n) (D) O(n) (E) O(n log n)
10. In a Fibonacci heap of size n, the worst case time complexity of n delete-min operations is
(A) O(n) (B) O(n log*n) (C) O(n log n) (D) $O(n^2)$ (E) $O(n^2 \log n)$
11. In a Fibonacci heap of size n, the worst case time complexity of n decrease-key operations is
(A) O(n) (B) O(n log*n) (C) O(n log n) (D) $O(n^2)$ (E) $O(n^2 \log n)$
12. In a leftist heap of size n, the worst case time complexity of one insert operation is
(A) O(1) (B) O(log*n) (C) O(log n) (D) O(n) (E) O(n log n)
13. In a leftist heap of size n, the worst case time complexity of one find-min operation is
(A) O(1) (B) O(log*n) (C) O(log n) (D) O(n) (E) O(n log n)
14. An AVL tree of 100 nodes has height at most
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9
15. A 2-3-4 tree of 255 nodes has height at most
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7
16. In a binomial heap of 2020 items, the maximum order among all binomial trees is
(A) 8 (B) 9 (C) 10 (D) 11 (E) 12
17. What is the size of the smallest Fibonacci heap of order 6?
(A) 8 (B) 16 (C) 21 (D) 32 (E) 64

見背面

二、(32%)複選題（每題 4 分，每答錯一個選項倒扣 0.8 分至該題 0 分止）

18. Which of the following statements are true?
    (A) The search operation in a binary search tree of size n is $O(\log n)$.
    (B) The height of a binary search tree of size n is $\Omega(\log n)$.
    (C) The search operation in an AVL tree of size n is $O(\log n)$.
    (D) The height of an AVL tree of size n is $O(\log n)$.
    (E) The delete operation in an AVL tree of size n is $O(\log n)$.
19. Consider the AVL tree in Fig.1.
    (A) In Fig.1, after inserting the key 66, then the parent of key 66 is key 61.
    (B) In Fig.1, after inserting the key 99, then the key 95 has two children.
    (C) In Fig.1, after inserting the key 30, then the parent of key 30 is key 35.
    (D) In Fig.1, after deleting the key 75, then the key 87 has two children.
    (E) In Fig.1, after deleting the key 1, then the parent of key 23 is key 13.
20. In a red-black tree of size n
    (A) The root is always black.
    (B) Each node is colored red or black.
    (C) No root-to-external-node path has two consecutive red nodes.
    (D) All root-to-external-node path have the same number of black nodes.
    (E) The height of is $O(\log n)$.
21. Consider the red-black tree in Fig.2
    (A) In Fig.2, after top-down inserting the key 86, then the color of key 86 is red.
    (B) In Fig.2, after top-down inserting the key 9, then the parent of key 9 is key 13.
    (C) In Fig.2, after top-down deleting the key 87, then the color of key 85 is red.
    (D) In Fig.2, after top-down deleting the key 10, then the parent of key 13 is key 15.
    (E) The 2-3-4 tree representation of Fig.2 has height 2.
22. Consider the AA tree in Fig.3
    (A) Key 4 and key 63 have the same level.
    (B) Key 62 and key 69 have the same level.
    (C) In Fig.3, after inserting the key 1, the key 4 is in the same level of key 42.
    (D) In Fig.3, after inserting the key 61, the parent of key 61 is key 62.
    (E) In Fig.3, after inserting the key 88, the parent of key 86 is key 77.
23. Which of the following operations are supported by hash table?
    (A) Find key
    (B) Insert key
    (C) Delete key
    (D) Find-Min
    (E) Delete-Min
24. Which of the following statements are correct?
    (A) In a binomial heap, there is at most one binomial tree of order 3.
    (B) In a binomial heap of n items, the height among all binomial trees is $O(\log n)$.
    (C) In a binomial heap of n items, there is $O(\log n)$ binomial trees.
    (D) Stack is first-in-first-out.
    (E) Queue is first-in-first-out.
25. Let $f(n) \sim g(n)$ denote $f(n) \in \Theta(g(n))$, and $f(n) \succ g(n)$ denote $f(n) \in \Omega(g(n))$ but $f(n) \notin \Theta(g(n))$. Which of the following statements are correct?
    (A) $\lceil log_2 n \rceil ! \sim n \log n \succ \log n \sim log_e n \succ 10000^{10000}$.
    (B) $n^n \succ n! \sim (\log n)^n \succ 4^n \succ n^3 2^n$.
    (C) $n! \succ (\log n)^n \succ \log(n!) \sim n \log n \succ n + 2^{log_3 n}$.
    (D) If $f(n)$ is $O(\sqrt{n})$, then $f(n)^2$ is $O(n)$.
    (E) If $f(n)$ is $O(n)$, then $2^{f(n)}$ is $O(2^n)$.

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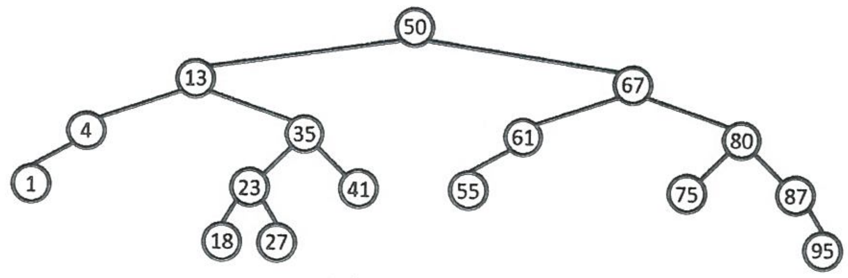

Fig.1 AVL tree

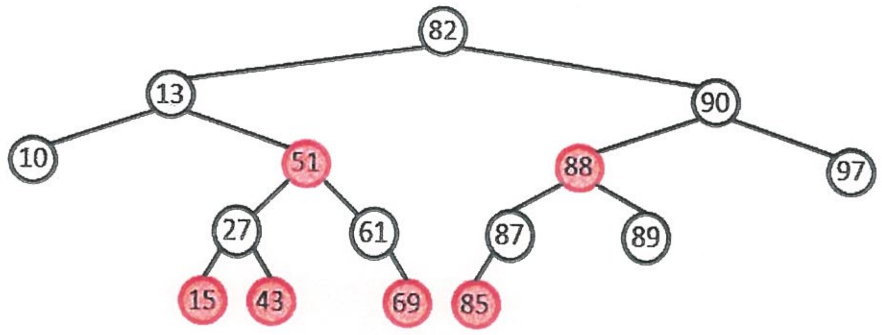

Fig.2 Red-black tree

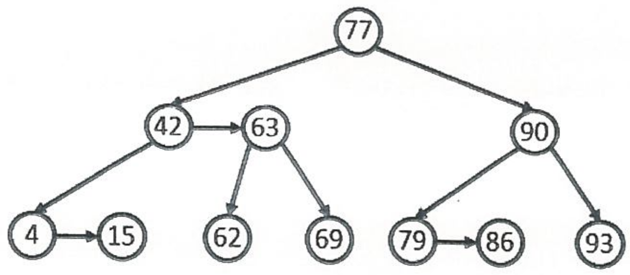

Fig.3 AA tree