第二大題選擇題(II. Multiple Choice Questions)考生應作答於『答案卡』

I. Statistical Case Analysis (50%)（每題 5 分）

Professor Samuel Oak investigates a special pocket monster, Pikachu, for its feeding and weight. Professor Oak randomly samples 12 Pikachu monsters and obtains data below for regression analysis, which is specified as $Weight_i = a_0 + a_1 Feed_i + a_2 Gender_i + u_i$. *Weight* and *Feed* are weight of Pikachu and forage for Pikachu per day. *Gender* is a binary variable, with value 1 for male, and zero for female. The regression coefficients are estimated by OLS. Professor Oak is aware of assumptions for multiple linear regression analysis below.

MLR 1: Regressions are linear in parameters
MLR 2: Random sampling
MLR 3: Non-perfect collinearity among independent variables
MLR 4: $Corr(u, x_j)=0, j=1 \dots k$.
MLR 5: $Var(u|x_1 \dots x_k)=\sigma^2$
MLR 6: Normality of u_i

Feed	1	1	1	1	1	2	2	2	3	3	3	3
Weight	4	4.5	5	5.5	6	6	6	6.5	6.5	7	7.5	8
Gender	0	0	0	1	1	0	1	0	1	1	0	1

1. Supposed that the dynamic of *Weight* could be theoretically defined by random variable y, where y=*Weight*/10; $f(y) = 1$, and $0 < y < 1$. Given another random variable z, and $f(z) = 1$, and $0 < z < 1$; y and z are independent. Please find $f(y|y < z)$.
2. Please use Stem and Leaf Plots to illustrate the frequency of *Weight* from the given data.
3. Please use ANOVA to analyze the relationship between *Feed* and *Weight* for Professor Oak. Write down your testing hypothesis and tabulate the ANOVA table. (Note: $F_{\alpha=0.05, (2,9)}=4.26$)
4. State the validation of the linearity assumption required in OLS (MLR1)
5. Verify the non-perfect collinearity for MLR 3 by showing centered and uncentered Pearson correlation coefficients. (1+4 points)
6. To test heteroskedasticity, Professor Oak groups the sample into two subgroups: Feed>1 and Feed ≤1. Please help Oak tests (MLR 5) by computing *F*-test. (Note: $F_{\alpha=0.05, (4,6)}=4.53$)
7. Verify the normality assumption required in OLS (MLR6).
8. Please estimate coefficient a_1 and test null hypothesis H_0: $a_1 = 0$ by *t*-test. (2+3points)
9. To test exogeneity assumption, i.e., $Corr(u, x_j)=0$, Professor Oak plans to compute Pearson correlation coefficients of estimated regression residuals and explanatory variables: $corr(\hat{u}, Feed)$ and $corr(\hat{u}, Gender)$. He would expect that two correlation coefficients are closed to zero under MLR 4. Please help Oak and complete the test.
10. From previous question, do you think his way to test exogeneity assumption a good method? Why or why not? Please state your reasons in 1 sentences.

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

II. Multiple Choice Questions (50%)（每題 5 分）

1. Below is the joint probability distribution of economic conditions and stock performance. Suppose $W = 4 + 8X$. What is the value of E(W)?

	Recession (X=0)	Boom (X=1)	Total
Negative returns (Y=0)	0.15	0.07	0.22
Positive returns (Y=1)	0.15	0.63	0.78
Total	0.30	0.70	1.00

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(a) 4.3
(b) 5.9
(c) 7.5
(d) 9.6

2. $Y_i, i = 1, \ldots, n.$ are i.i.d. Bernouli random variables with $p = 0.3$. Let $\bar{Y}$ denote the sample mean. Use the central limit theorem to compute approximations for $Pr(\bar{Y} \geq 0.38)$ when $n = 100$.
(a) 0.04
(b) 0.05
(c) 0.43
(d) 0.57

3. X and Z are two independently distributed standard normal random variables. $Y = X^2 + 5Z$. Please calculate E(Y).
(a) -1.5
(b) 1
(c) 0.5
(d) 2.5

4. Suppose that a researcher, using data on corporate earnings and capital expenditure from 1000 companies, estimate the OLS regression:
$\widehat{Earnings} = 134 + 4.24 \times Capital\_Expenditure$
All regression variables are measured in million dollars. The sample average capital expenditure across the 1000 firms is 43 million dollars. What is the sample average of the corporate earnings across the 1000 firms?
(a) 134 million dollars
(b) 286 million dollars
(c) 316 million dollars
(d) It cannot be determined.

5. Consider a regression $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i$. All OLS assumptions are satisfied. You are interested in β_1, which is the causal effect of X_1 on Y. Suppose X_1 and X_2 are uncorrelated. You estimate β_1 by regressing Y on only X_1, but not X_2. Which of the following statement is correct regarding the OLS (ordinary least squares) estimator $\hat{\beta}_1$ in the regression you have estimated?
(a) It is consistent and unbiased.
(b) It is consistent but biased in small sample.
(c) It is inconsistent and biased.
(d) We cannot be certain whether it is consistent.

Please use the following information to answer questions 6 and 7.
A random sample of 100 MBA students is selected from a population and these students' weight and height are recorded. A regression of weight (measured in pounds) on height (measured in inches) yields
$\widehat{Weight} = -50 + 3.21 \times Height, R^2 = 0.57, SER = 11,$

where SER refers to the standard error of the regression. Suppose now we change the unit of measurement from pounds to kilograms for weight and from inches to centimeters for height. (Note: 1 pound equals 0.454 kilogram, and 1 inch equals 2.54 centimeters.)

6. What is the estimated regression coefficient of $Height$ in the new regression model?

(a) 17.959
(b) 8.153
(c) 3.21
(d) 0.574

7. What is the estimated regression R^2 in the new regression model?

(a) 0.26
(b) 0.45
(c) 0.57
(d) 0.69

8. The interpretation of the slope coefficient in the model $\ln(Y_i) = \beta_0 + \beta_1 X_i + u_i$ is as follows:

(a) a 1% change in X_i is associated with a β_1% change in Y_i.
(b) a 1 unit change in X_i is associated with a change in Y_i of β_1%.
(c) a 1% change in X_i is associated with a β_1 unit change in Y_i.
(d) a 1 unit change in X_i is associated with a change in Y_i of $\beta_1 \times 100\%$.

9. The true population model is $Y_i = \beta_0 + \beta_1 X_i + u_i$. Suppose that you estimate the following model by OLS (ordinary least squares): $\tilde{Y}_i = \gamma_0 + \gamma_1 X_i + \varepsilon_i$, where $\tilde{Y}_i$ is the estimate of Y_i, and $\tilde{Y}_i$ equals $Y_i + w_i$. Assume that w_i is not correlated with X. $\hat{\gamma}_1$ is the OLS estimator of γ_1. Which of the following statements is correct?

(a) The OLS estimator $\hat{\gamma}_1$ is a consistent estimator of β_1.
(b) The OLS estimator $\hat{\gamma}_1 < \beta_1$ in a large sample.
(c) $E(\hat{\gamma}_1) > \beta_1$ in a small sample.
(d) $E(\hat{\gamma}_1) < \beta_1$ in a large sample.

10. An ordinary light bulb has a mean life of 835 hours and a standard deviation of 100 hours. Jennifer is developing a new manufacturing process through which she hopes can improve the average life of a light bulb. She randomly selects 500 light bulbs produced by the new process. She says that she will believe that the new process works better than the old process if the sample mean life of the 500 light bulbs is greater than 840 hours; otherwise, she will conclude that the new light bulbs are no better than the old ones. Let μ denote the mean life (in hours) of the new light bulb. Consider the null and alternative hypotheses H_0: $\mu =$ 835 and H_1: $\mu =>$ 840. What is the size of the plant manager's testing procedure? Please use Central Limit theorem to approximate the distribution of the t-statistics.

(a) 0
(b) 0.065
(c) 0.13
(d) 0.26

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Appendix: Z-Table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002

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