1. Let $A=\begin{bmatrix}1 & 2 & 0 & 1\\ 1 & 2 & 1 & 1\\ r_1 & r_2 & r_3 & r_4\\ s_1 & s_2 & s_3 & s_4\end{bmatrix}$. Suppose both the last two rows are linear combinations of the first two. Your answers to some of the following may involve the r_i and s_i.
   (a) Find a basis for the row space of A. (5%)
   (b) Find a basis for the null space of A. (5%)
   (c) Find a basis for the column space of A. (5%)
   (d) Find the determinant of A^{99}. (5%)

2. Suppose v is an eigenvector for an $n \times n$ matrix A with eigenvalue λ.
   (a) Show that cv is also an eigenvector with eigenvalue λ for any $c \neq 0$. (5%)
   (b) Show v is also an eigenvector for A^2 with eigenvalue λ^2. (5%)

3. Suppose you have a dynamical system given by

$$x(t+1) = x(t) + 2y(t)$$
$$y(t+1) = 4x(t) + 3y(t)'$$

   with initial conditions $x(0) = 2$ and $y(0) = 1$. Find explicit formulas for $x(t)$ and $y(t)$. (20%)

4. Determine whether the set of polynomials is linearly independent or linearly dependent (20%):

$$p_1(x) = 1, \quad p_2(x) = -2 + 4x^2, \quad p_3(x) = 2x, \quad p_4(x) = -12x + 8x^3$$

5. The 3 × 3 matrix A satisfies

$$A\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & -2\\ 1 & 1 & 2\end{bmatrix} = \begin{bmatrix}2 & 0 & 1\\ 2 & 1 & -2\\ 1 & 1 & 2\end{bmatrix}\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 3\end{bmatrix}$$

   (a) What are the eigenvalues of A? (5%)
   (b) Find a basis consisting entirely of eigenvectors of A. (5%)
   (c) Find an orthogonal matrix Q such that Q^TAQ is diagonal. (10%)

6. Let $S: \mathbb{R}^2 \to \mathbb{R}^2$ be a transformation given by $S(x,y) = (1-xy, x+y)$. Determine whether S is a linear transformation. Explain. (10%)

試題隨卷繳回