1. Let $X_1, \dots, X_n$ be a random sample from the Normal distribution with mean θ and variance θ^2, i.e. $X_i \overset{iid}{\sim} N(\theta, \theta^2)$
   (a) (10 points) Find the maximum likelihood estimator (MLE) of θ, if it exists.
   (b) (5 points) Let $T_1 = \bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$ and let $T_2 = c_n S = c_n\sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}}$.
   Find the constant c_n such that T_2 is an unbiased estimator of θ.
   (c) (5 points) Consider the estimator of θ of the form: $W(\alpha) = \alpha T_1 + (1-\alpha)T_2, 0 \le \alpha \le 1$.
   Find the mean square error (MSE) of $W(\alpha)$ in terms of $Var(T_1)$, $Var(T_2)$, and α.
   (d) (5 points) Assume that $Var(T_2) \approx \frac{\theta^2}{2n}$. Find the value of α that reaches the smallest MSE.
2. Let $X_1, \dots, X_n$ be a random sample from the distribution with pdf
   $$f(x|\theta) = \theta x^{\theta - 1}, 0 < x < 1, \theta > 0.$$
   (a) (5 points) Show that the random variable $W = -\log(X)$ is an exponential distribution and find a method of moments estimator of θ.
   (b) (5 points) Find the uniform minimum variance unbiased estimator (UMVUE) of $1/\theta^2$. Be sure to justify your answer.
   (c) (5 points) Find the Cram'er-Rao lower bound for the variance of unbiased estimator of $1/\theta^2$.
3. Let $X_1, \dots, X_n$ be a random sample from a uniform distribution $U(0, \theta)$.
   (a) (5 points) Let $Y = \max(X_1, X_2, \dots, X_n)$. Find the distribution of Y/θ.
   (b) (5 points) Find the $(1-\alpha) \times 100\%$ confidence interval for θ.
4. Testing the value of the parameter p of a Bernoulli distribution with 5 trials. Let X be the number of successes, then $X \sim B(5, p)$. To test $H_0: p = 0.5$ $vs.$ $H_A: p = 0.2$
   (a) (2 points) When $X \in \{0, 1\}$, reject H_0. Find the significant level α.
   (b) (2 points) Find the probability of type II error.
   (c) (2 points) Find the power of the test.
   (d) (2 points) When hypothesis test: $H_0: p = 0.5$ $vs.$ $H_A: p < 0.5$ with the same significant level α given in Question(a), find the rejection region.
   (e) (2 points) When hypothesis test: $H_0: p = 0.5$ $vs.$ $H_A: p < 0.5$ with the same significant level α given in Question(a), find the $\inf\{P(type\ II\ error\ |H_A)\}$.
5. True or False
   (a) (1 point) In hypothesis test, $H_0 \cap H_A \ne \emptyset$, were $\emptyset$ is empty set.
   (b) (1 point) In hypothesis test, significance level can be set as any value ranged by (0, 1).
   (c) (1 point) To test whether the means of K populations are equal, if we have 1st and 2nd populations have significant different means by t-test, the F-test of one-way ANOVA will also reject H_0.
   (d) (1 point) In one-way ANOVA, the denominator and the nominator of F test statistic are independent.
   (e) (1 point) In hypothesis test, "do not reject H0" is equivalent to "H0 is true".
6. $X_1, X_2, \dots, X_n \overset{iid}{\sim} Poi(\theta), where\ P(X = x) = \frac{\theta^x}{x!}e^{-\theta}, x = 0, 1, 2, \dots$. To test $H_0: \theta = \theta_0$ vs $H_A: \theta > \theta_0$
   (a) (5 points) Using Neyman-Pearson Lemma to find the UMP test with significant level α.
   (b) (5 points) Find the generalized likelihood ratio test with significant level α.
7. Two populations have equal variance and have means μ_1 and μ_2, separately. We have random sample $(X_{11}, X_{21}, \dots, X_{n_1 1})$ from population 1 and random sample $(X_{12}, X_{22}, \dots, X_{n_2 2})$ from population 2.
   There are two methods to test $H_0: \mu_1 = \mu_2$ vs $H_0: \mu_1 \ne \mu_2$, one is t-test of two unpaired populations, the other is F test of one-way ANOVA.
   (a) (6 points) Separately write down the test statistics of t-test and F-test.
   (b) (2 points) Which distributions do the test statistics of t-test and F-test separately follow?
   (c) (7 arbitrarily points) Show that $T^2 = F$, where T is the test statistics of t-test and F is the test statistic of F-test. (show the deriving detail)

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8. If gene frequencies are in equilibrium, the genotypes AA, Aa, and aa occur in a population with frequencies $p_{11} = \theta^2$, $p_{12} + p_{21} = 2\theta(1-\theta)$, and $p_{22} = \theta^2$, according to the Hardy Weinberg law.
For a 2×2 contingency table with observed counts$(n_{11}, n_{12}, n_{21}, n_{22})$:

	A	a	
A	$n_{11}(p_{11})$	$n_{12}(p_{12})$	$n_{1+}(p_{1+})$
a	$n_{21}(p_{21})$	$n_{22}(p_{22})$	$n_{2+}(p_{2+})$
	$n_{+1}(p_{+1})$	$n_{+2}(p_{+2})$	

(a) (4 points) If $(n_{11}, n_{12}, n_{21}, n_{22}) \sim Multinormal(n_{++}, p_{11}, p_{12}, p_{21}, p_{22})$, find the MLE of θ (in terms of $n_{11}, n_{12}, n_{21}, n_{22}$).
(b) (3 points) Using Chi-square test to test $H_0: \theta = \theta_0$ vs. $H_A: not\ H_0$. What is the statistic test?
(c) (3 points) What are the dimensions of H_0 and H_A, separately? What is the degree of freedom of Chi-square test in Question(b)?

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