

1. Set  $f(x) = 3/5 x^{5/3} - 3x^{2/3}$ . Find (a) the intervals on which  $f$  increases or decreases, and (b) the intervals on which the graph of  $f$  is concave up and the interval on which it is concave down. Determine whether the graph of  $f$  has vertical tangents or vertical cusps. Sketch the graph of  $f$ . (20%)
2. Assume that  $f$  is a continuous function and that  $\int_0^x tf(t)dt = \sin x - x \cos x$ . (a) Determine  $f(\pi/2)$ . (b)  $f'(x)$ . (10%)
3. The integral  $\int_0^{\infty} \frac{dx}{\sqrt{x(1+x)}}$  is improper. Show this integral is convergent and evaluate this integral. (15%)
4. Find the limit.  $\lim_{x \rightarrow \infty} (\cosh x)^{1/x}$  (10%)
5. Expand  $g(x) = \cos x$  in powers of  $x - \pi$ , give the Lagrange form of the remainder, and specify the values of  $x$  for which the expansion is valid. (15%)
6. Find the directional derivative of  $f(x, y, z) = xe^{y^2 - z^2}$  at  $(1, 2, -2)$  in the direction of the path  $\vec{r}(t) = t\vec{i} + 2\cos(t-1)\vec{j} + 2e^{t-1}\vec{k}$ . (10%)
7. Evaluate the triple integral  $\iiint_T (xyz)^2 dx dy dz$ , where  $T$  is the solid bounded by the plane  $z = y + 1$ ,  $y + z = 1$ ,  $x = 0$ ,  $x = 1$ ,  $z = 0$ . (10%)
8. Find the total flux out of the solid with  $\mathbf{v} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$  and the cylinder  $x^2 + y^2 \leq 4$ ,  $0 \leq z \leq 4$ , including the top and base. (10%)

試題隨卷繳回