1. (10%) Given a position vector below, determine the velocity, speed, acceleration, the tangential and normal component of acceleration, the curvature, and the unit tangent vector, unit normal vector.

$\mathbf{F}=3t\mathbf{i}-2\mathbf{j}-t^2\mathbf{k}$

2. (10%) Find the Laplace transform of $f(t)=[\sin(t)-\cos(t)]^2$

3. (10%) Solve the differential equation, $y''-ty'+y=1$, $y(0)=1$, $y'(0)=2$

4. (10%) Produce a matrix **P** that diagonalizes the matrix below:

$$\begin{pmatrix} -7 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -4 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

5. (10%) Find the general solutions of $y''-6y'+9y=6x+2-12e^{3x}$

6. (25%) Let $u=u(x,y,t)$ be a function of the space (x,y) and the time t. $S=S(x,t)$ is a function of x and t, and c_1 and c_2 are constant. Determine whether the method of separation of variables is applicable to the following partial differential equations or not (each has 5 %). The rationale behind your answer needs be given; otherwise, it will not be credited.
(a) $\dfrac{\partial u}{\partial t}=c_1\sin(u)+c_2\dfrac{\partial^2 u}{\partial x^2}$, (b) $\dfrac{\partial u}{\partial t}=c_1e^x\dfrac{\partial^2 u}{\partial x^2}+c_2y^2\dfrac{\partial^2 u}{\partial y^2}$, (c) $\dfrac{\partial u}{\partial t}=c_1\dfrac{\partial u}{\partial x}\dfrac{\partial u}{\partial y}+c_2e^x\dfrac{\partial^2 u}{\partial y^2}$, (d) $\dfrac{\partial u}{\partial t}=c_1\dfrac{\partial^2 u}{\partial x^2}+c_2\dfrac{\partial^2 u}{\partial y^2}+S(x,t)$, (e) $\sin(t)\dfrac{\partial^2 u}{\partial t^2}=c_1\dfrac{\partial u}{\partial x}+c_2\dfrac{\partial^2 u}{\partial y^2}$.

7. (25%) Let $u=u(x,y)$ be a function of x and y. $f(x)$, $g(x)$, $f_1(y)$, and $g_1(y)$ are given; K and L are positive constant. Solve the problem:

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0, \quad 0<x<K, \quad 0<y<L \\ u(x,0)=f(x), \quad u(x,L)=g(x), \quad 0<x<K \\ u(0,y)=f_1(y), \quad u(K,y)=g_1(y), \quad 0<y<L \end{cases}$$

試題隨卷繳回