國立臺灣大學 109 學年度碩士班招生考試試題

題號： 101


1. (5%) (5%) Let $X$ have a Gamma distribution with parameters $\alpha$ ($> 2$) and $\beta$. Compute the mean and variance of $1/X$.

2. (15%) Let the distribution of $U$ conditioning on $T = t$ be Uniform$(0, t)$ and $T$ follow an exponential distribution with rate $\lambda > 0$. Derive the probability density function of $U$.

3. (15%) Let $X_1, \ldots, X_n$ be a random sample from a continuous distribution $F(x)$ and $X_{(1)}, \ldots, X_{(n)}$ be the order statistics of $X_1, \ldots, X_n$. Derive the joint distribution of $F(X_{(i)})$ and $F(X_{(j)})$ for $1 \le i < j \le n$.

4. (15%) (10%) Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution on $[0, \theta]$. Derive the distributions of the moment and maximum likelihood estimators of $\theta$.

5. (15%) Let $X_1, \ldots, X_n$ be a random sample from $Poisson(\lambda)$ and $\lambda$ have $Gamma(\alpha, \beta)$, where $\alpha$ and $\beta$ are known positive constants. Find the Bayes estimator of $\lambda$ based on the loss function $L(\lambda, \delta(X_1, \ldots, X_n)) = |\delta(X_1, \ldots, X_n) - \lambda|$

6. (20%) Let $X_1, \ldots, X_{n+1}$ be a random sample from Bernoulli$(\pi)$ and $h(\pi) = P(\sum_{i=1}^{n} X_i > X_{n+1}|\pi)$. Find the asymptotic distribution of the uniformly minimum variance unbiased estimator of $h(\pi)$.

試題隨卷繳回