

1. (8%) Find the limit: $\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x}$.
2. (8%) Find the slope of a tangent line to the circle $x^2 + y^2 = 5x + 4y$ at the point $x = 5, y = 4$.
3. (8%) Evaluate $\int_0^1 x e^{2x} dx$.
4. (10%) Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$.
5. (8%) Find the gradient $\nabla f(x, y)$ for $f(x, y) = x^2 y + y^3$.
6. (8%) Find the integral $\int_0^2 \int_{y^2}^4 1 dx dy$.

線性代數

7. (15%) Use Cramer's rule to find all solutions of the following system.

$$\begin{aligned} w + x + 3y - 2z &= 0 \\ 2w + 3x + 7y - 2z &= 9 \\ 3w + 5x + 13y - 9z &= 1 \\ -2w + x - z &= 0 \end{aligned}$$

8. (15%) Use the simplex method to solve the following standard linear programming problem. Maximize $z = 8x_1 + 9x_2 + 5x_3 + 10$ subject to

$$\begin{aligned} x_1 + x_2 + 2x_3 &\leq 2 & x_1 &\geq 0 \\ 3x_1 + 4x_2 + 6x_3 &\leq 5 & x_2 &\geq 0 \\ 7x_1 + 7x_2 + 4x_3 &\leq 10 & x_3 &\geq 0 \end{aligned}$$

9. (20%) Find an orthogonal matrix P such that $P^T A P$ is a diagonal matrix if

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$