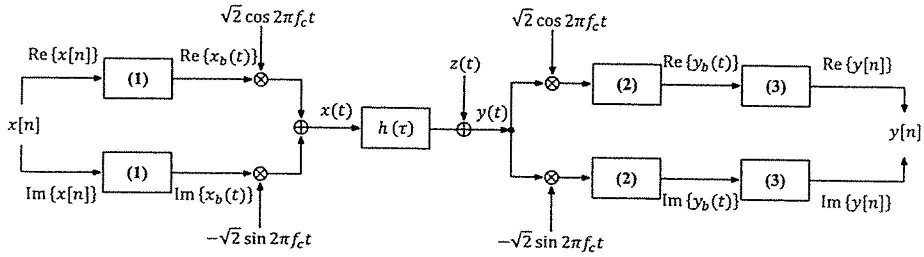
國立臺灣大學 104 學年度碩士班招生考試試題

1. [12] A digital passband communication system over a telephone channel uses ideal sinc pulses and signal at Nyquist rate. Let W denote the total available bandwidth, f_c denote the carrier frequency, and $h(\tau)$ denote the impulse response of the LTI telephone channel. Let $\frac{N_0}{2}$ denote the spectral density of the additive white Gaussian noise process $\{z(t)\}$.

   (a) [4] Complete the system diagram of this digital baseband communication system shown below, by filling in the missing components (1), (2), and (3) in your answer sheets.



   (b) [8] Write down the equivalent discrete-time baseband channel model that describes the relationship between $x[n]$ and $y[n]$. You should specify all components in your model by the parameters given above.

2. [16] Consider BPSK and QPSK modulation techniques over the discrete-time baseband channel $y[n] = x[n] + z[n]$, where $z[n] = z_I[n] + jz_Q[n]$ and $z_I[n], z_Q[n]$ are iid Gaussian with zero mean and variance $\frac{\sigma^2}{2}$. In other words, $z[n]$ is circular symmetric complex Gaussian. Let E_b denote the energy to send a single information bit.

   (a) [6] Compare the $\frac{E_b}{\sigma^2}$ performance of BPSK and QPSK over the channel, for the same **symbol error probability** P_e.
   (You can assume that P_e is small in making any approximations. You may find the approximation $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt \approx e^{\frac{-x^2}{2}}$ useful.)

   (b) [4] Suppose the in-phase and quadrature-phase components of each QPSK symbol carry two independent information bits. Repeat the comparison in part (a) between BPSK and QPSK but now for the same **bit error probability**.

   (c) [6] Repeat (a) with comparison between QPSK and 4-PAM.

3. [22] Consider the detection problem of x given y_1, y_2, where $y_1 = x + z_1, y_2 = x + z_1 + z_2$. Here, x is equally like to be $\sqrt{E}$ and $-\sqrt{E}$, and z_1, z_2 are iid standard Gaussian random variables independent of x.

   (a) [4] Find the optimal detector of x given y_1, y_2 and compute the probability of error.

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(b) [4] Consider now the optimal detector of x using y_1 only. Find the additional energy, if any, needed to achieve the same probability of error as the optimal detector in (a).

(c) [10] Now, suppose that $y_2 = z_1 + z_2$. Repeat (a) and (b) above.

(d) [4] Now, suppose that z_1, z_2 are dependent. Discuss how your answers in (a) – (c) will change according to the correlation coefficient between z_1 and z_2.

4. [15] Please answer the following problems clearly.
   (a) [3] What are the conditions required for a system to be a linear system ?
   (b) [3] What are the conditions required for a system to be a linear time-invariant (LTI) system ?
   (c) [3] For an LTI system with unit impulse response h(t), derive the convolution integral equation in terms of h(t) relating its input x(t) and output y(t).
   (d) [3] For a simple circuit with a resistor R and a capacitor C in cascade, an input voltage signal x(t), and the corresponding output capacitor voltage signal y(t), derive the differential equation describing the relationship between x(t) and y(t).
   (e) [3] For the simple RC circuit of (d), is it a causal linear system ? Why ?

5. [18] Consider a causal LTI system h(t) with a rational system function H(s) of two poles at s = −2 and s = 4. The output of the system with input x(t) = 1 is given by y(t) = 0. Moreover, the unit impulse response h(t) = 4 at $t = 0^+$.
   (a) [8] Find the corresponding system function H(s).
   (b) [5] Show the region of convergence of H(s).
   (c) [5] Is the system stable ? Why ?

6. [11] Consider a causal and stable LTI system with a rational system function H(z) of a pole at z = 1/2 and a pole on the unit circle. Let h[n] be its unit impulse response.
   (a) [4] Does the Fourier transform of $(1/2)^n h[n]$ converge ? Why ?
   (b) [3] Does h[n] have finite duration ? Why ?
   (c) [4] We create a new system with the unit impulse response f[n] = n{h[n]*h[n]}, where * denotes the convolution operation. Is the new system causal and stable ? Why ?

7. [6] The samples x(nT), n = 0, ±1, ±2, …, of a continuous-time signal x(t) are taken at a sampling frequency $\omega_s = 2\pi/T > 2\omega_M$, where ω_M denotes the highest frequency of x(t). Derive the equation for reconstructing the original x(t) from its samples x(nT) by using an ideal low-pass filter.

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