1. (15%) Let

$$x[n]=\begin{cases}1, & 0\le n\le 9\\ 0, & \text{elsewhere}\end{cases} \quad \text{and} \quad h[n]=\begin{cases}1, & 0\le n\le N\\ 0, & \text{elsewhere}\end{cases},$$

where $N \le 9$ is an integer. Determine the value of N, given that $y[n] = x[n] * h[n]$ and $y[4] = 5$, $y[14] = 0$. ($y[n]$ is the convolution of $x[n]$ and $h[n]$.)

2. The differential equation

$$\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+3y(t)=\frac{dx(t)}{dt}+2x(t),$$

(a) (15%) Determine $h(t)$ by using Fourier transform of H(jω), given that $y(t) = x(t) * h(t)$.

(b) (20%) If $x(t)=e^{-t}u(t)$, determine $y(t)$ by using Fourier transform of Y(jω), given

$u(t)=1, t\ge 0\,;\, u(t)=0, t<0.$

3. (25%) Given that $A=\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}$, please compute e^A. (Hint: use Cayley-Hamilton theorem and Taylor series expansion for e^A)

4. (25%) Given a differential equation $U\frac{d}{dx}\left(x^2U\right)=C$, where U is the dependent variable, x is the independent variable and C is a constant. The initial condition is that at $x=x_0, U=U_0$.

Please derive the solution in the form of $U^2=U_0^2(\ldots)+C(\ldots)$

(Hint: use an integration factor to solve the differential equation)

試題隨卷繳回