(1) (20%) Consider the outflow of water from cylindrical tank with a hole at the bottom as shown in Figure 1. Please find the height of water in the tank at any time if the tank has diameter 2m, the hole has diameter 1 cm, and the initial height of the water when the hole is opened is 2.25m. (Assuming the outflowing water has velocity $v(t) = 0.3\sqrt{2gh(t)}$, where $h(t)$ is the height of the water at time t, and g=980 cm/sec^2 is the acceleration of gravity.)

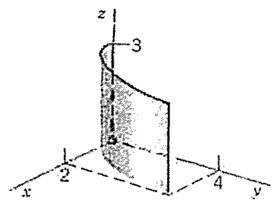
(2) (10%) (a)Find the solution of the following wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad 0 < t < \infty, \quad ICs.\begin{cases} u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}$$

(10%) (b)If $f(x) = 0$, $g(x) = x$, $0 \le x \le 1$, find the solutions of $u(-\frac{1}{2}, \frac{2}{3})$, and $u(\frac{1}{3}, \frac{1}{6})$

(3) (20%) Compute the flux of water through the parabolic cylinder S:

$y = x^2, 0 \le x \le 2, 0 \le z \le 3$ (as shown in Figure) if the velocity vector is $\vec{v} = \left[3z^2 \quad 6 \quad 6xz\right]$, speed being measured in m/sec.



(4) (20%) Find the Fourier integral of Delta function $\delta(t-a)$, and then using the result to evaluate the Fourier transform of $A\sin\beta t$. a and β are constants.

$$\left[\ \text{hint:}\ F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\cdot e^{-i\omega t}dt,\ \ f(t) = \int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega\ \right]$$

(5)(20%) Find a matrix $\mathbf{P}$ such that $\mathbf{P}^T\mathbf{A}\mathbf{P} = \mathbf{D}_\lambda$, where $\mathbf{D}_\lambda$ is a diagonal matrix formed by the eigenvalues of $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 5 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 5 \end{bmatrix}.$$