※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

1. (25%) A composite circular shaft of length $L = 2$ m and fixed on both ends is made of two materials with shear modulus $G = 2$ GPa for radius $0.08 \leq r \leq 0.1$ m and $G = 1$ GPa for radius $0 \leq r \leq 0.08$ m. The shaft is subjected to a distributed torsional moment load $t = 31$ kN-m/m on the left half span $0 \leq x \leq 1$ m but no load on the right half span $1 \leq x \leq 2$ m.
   (a) (10%) Find the reaction torques at the two ends. Draw the torque diagram for the shaft.
   (b) (5%) Determine the shear strain distribution over the cross section near the left end $x = 0$. Do the shear strains show the shear deformation for the axial-radial coordinate plane? or the shear deformation for the axial-circumferential coordinate plane? or the shear deformation for the plane of cross section?
   (c) (5%) Determine the shear stress distribution over the cross-sections near the left end.
   (d) (5%) Calculate the maximum twist angle and the location at which it occurs.

2. (25%) A horizontal cantilever beam of length 1 m ($0 \leq x \leq 1$ m) is fixed at the left end $x = 0$ and acted upon by a concentrated load $P = 1$ kN (vertical downward in the positive Z-direction) at the right end $x = 1$ m. The beam is made of a thin-walled unequal-leg angle section which has one vertical leg ($0 \leq Z \leq 100$ mm, $0 \leq Y \leq 10$ mm) and one horizontal leg ($0 \leq Y \leq 40$ mm, $0 \leq Z \leq 10$ mm). Here xYZ is a right-handed rectangular coordinate system, of which the x-axis passes through the centroids of the angle sections. There are also the y-axis and z-axis parallel to the Y-axis and Z-axis, respectively, and intersecting at the centroid o of the angle section.
   (a) (5%) Locate the centroid o and the shear center S of the angle section.
   (b) (5%) Find the principal moments of inertia $I_{y_p} = I_{max}$, and $I_{z_p} = I_{min}$, and the angle θ_p between the y-axis and the y_p-axis (*i.e.* the major axis).
   (c) (10%) Should the end load P be applied at the centroid o or at the shear center S in order to avoid twisting? If P is applied at S, find the distributions of shear and normal stresses over the end section $x = 1$ m.
   (d) (5%) Find the (magnitude and direction of) deflection at the end $x = 1$ m.

3. (16%) Consider isotropic, linearly elastic materials with Young's modulus E, Poisson's ratio ν, shear modulus G, and thermal expansion coefficient α.
   (a) (8%) State the definitions of E, ν, G, and α. What kind of material tests can be used (and how) to measure them? What are their ranges of numeric values (with units)?
   (b) (2%) What is the relation between E, ν, and G?
   (c) (4%) Express the strain components σ_{ij} $(i, j = 1, 2, 3)$ in terms of the stress components σ_{ij} $(i, j = 1, 2, 3)$ (and the temperature T).
   (d) (2%) Do the principal axes of stresses and strains coincide? Why?

4. (9%) A beam of length L ($0 \leq x \leq L$) with both ends fixed is subjected to the transverse distributed load $q(x)$. Denote the normal and shear stresses induced in the beam by $\sigma(x, z)$ and $\tau(x, z)$, respectively. These stresses have resultants: the bending moment $M(x)$ and the shear force $V(x)$. The beam thus undergoes the deflection $w(x)$, slope $w'(x)$, curvature $\kappa(x)$, and axial strain $\epsilon(x, z)$. What is the elastic energy U stored in the beam in terms of stress? What is the elastic energy U stored in the beam in terms of moment? What is the work W done by the load $q(x)$? Is $U = W$?

5. (25%) Consider a prismatic member (rod/beam/column) of length L ($0 \leq x \leq L$) under a centric axial force P.
   (a) (15%) What are the key different consequences for the case P is tensile and for the case P is compressive? For each case draw a free-body diagram of a differential element of length dx of the member and indicate all forces/moments involved, and on the basis of the drawing, for each case, derive the equilibrium equations and the governing equations for displacements.
   (b) (10%) What roles do geometric quantities of cross sections (for example, cross sectional area A, moments of inertia I_{min} or I_{max}, *etc.*) play? How significant are end supports in influencing the load-carrying capacity? How significant are intermediate supports in influencing the load-carrying capacity?