國立臺灣大學 104 學年度碩士班招生考試試題

1.(20%) By using the Laplace transform method to solve the following Volterra integral equations of the second kind:

$$\text{(a)}\quad y(t) - \int_0^t \sin(t-\tau)y(\tau)d\tau = t,$$

$$\text{(b)}\quad y(t) - \int_0^t (1+\tau)y(t-\tau)d\tau = 1 - \sinh t.$$

2.(10%) Let $\mathbf{F} \in \mathbb{R}^3$ be a vector function given by

$$\mathbf{F} = z\mathbf{i} + x^3\mathbf{j} - (z+1)\mathbf{k},$$

and S a surface given by

$$S: \quad z = 4 - x^2 - y^2 \ \text{ between } \ z=0 \ \text{ and } \ z=4;$$

can you evaluate the following result:

$$\int\int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}$$

by using the Stokes' Theorem, where $\mathbf{n}$ is the outer normal of the surface S.

3.(10%) Let P, Q be continuously differentiable in the plane of (x,y) with $\partial Q/\partial x = \partial P/\partial y$ except at the points $(2,0)$, $(0,0)$, and $(-2,0)$. Let Γ_1 denote the circle $(x-1)^2+y^2=4$; let Γ_2 denote the circle $x^2+y^2=1$; and let Γ_3 denote the circle $(x+4)^2+y^2=9$. Further let

$$\oint_{\Gamma_1}(Pdx+Qdy)=10, \quad \oint_{\Gamma_2}(Pdx+Qdy)=20, \quad \oint_{\Gamma_3}(Pdx+Qdy)=5.$$

Find $\oint_\Gamma Pdx+Qdy$, where Γ is the circle $x^2+y^2=225$.

4.(10%) Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & 3 \end{bmatrix}.$$

Find the eigenvalues $\lambda_i,\ i=1,\ldots,4$ of $\mathbf{A}$. Can you find a non-singular matrix $\mathbf{P}$ to diagonalize $\mathbf{A}$.

5.(15%) By using the method of parameter variation to find the particular solution of

$$y''(x) + 4y(x) = \sec x, \quad -\frac{\pi}{4} < x < \frac{\pi}{4}.$$

6.(20%) Find the general solutions and singular solutions of

$$\text{(a)}\quad y(x) = xy'(x) - \frac{1}{4}[y'(x)]^2,$$

$$\text{(b)}\quad y(x) = xy'(x) + \frac{1}{y'(x)}.$$

7.(15%) Discuss the types of the following linear partial differential equation:

$$yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0,$$

and find the general solution of $u(x,y)$ when $x \neq y$.

試題隨卷繳回