

Problem 1. (25 points)

Consider the continuous bivariate probability distribution whose joint probability density function (pdf) is

$$f_{XY}(x, y) = 3(x^2 + y)/11 \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 1,$$

with $f(x, y) = 0$ elsewhere.

- (1) Find marginal pdf of X , $f_X(x)$.
- (2) For $0 \leq x \leq 2$, find the conditional pdf of Y given X .
- (3) Let $A = \{0 \leq Y \leq 1/2\}$. Find $Prob(A|x)$ for $x = 0, 1$.
- (4) For $0 \leq x \leq 2$, find conditional expectation function $E(Y|X)$.
- (5) Find $Var(X)$.

Problem 2. (15 points)

Suppose that (X, Y) is discrete, with $f_{XY}(x, y) = 1/3$ at each of three mass points, $(1, -1)$, $(0, 0)$ and $(1, 1)$.

- (1) Find $E(Y|X)$ and $E(X|Y)$.
- (2) Are X and Y independent? Why or why not.

Problem 3. (10 points)

Let $\{X_i\}$ be a sequence of *i.i.d.* *Bernoulli*(p) random variables, and let $N \sim Poisson(\lambda)$. Assume that $\{X_i\}$ is independent of N , and $S_N = \sum_{i=1}^N X_i$. Find $E(S_N)$ and $Var(S_N)$.

Problem 4. (30 points)

Suppose that $Y = a + bX + U$, where X and U are random variables and a and b are constants. Assume that $\mathbb{E}[U|X] = 0$ and that $Var(U|X) = X^2$.

- (a) Is Y a random variable? Why?
- (b) Is U mean independent of X ? Why?
- (c) Is U independent of X ? Why?
- (d) Show that $\mathbb{E}[U] = 0$ and that $Var(U) = \mathbb{E}[X^2]$.
- (e) Show that $\mathbb{E}[Y|X] = a + bX$ and that $\mathbb{E}[Y] = a + b\mathbb{E}[X]$.
- (f) Derive $Var(Y|X)$ and $Var(Y)$.

Problem 5. (20 points)

Consider the equation

$$rsp500_t = \beta_0 + \beta_1 pcipt_t + \beta_2 i3_t + u_t,$$

where the variable $rsp500$ is the monthly return on the Standard and Poor's 500 stock market index, at annual rate (this includes price changes as well as dividends), the variable $i3$ is the

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return on three-month T-bills, and $pcip$ is the percentage change in industrial production; these are also at an annual rate.

- (a) What signs do you think β_1 and β_2 should have? Explain briefly.

The estimated equation is

$$\widehat{rsp500}_t = 18.84 + 0.036 pcip_t - 1.36 i3_t$$

(3.27) (0.129) (0.54)

$$n = 557, R^2 = 0.012,$$

numbers in parentheses are standard errors.

- (b) Interpret the signs and magnitudes of the estimated coefficients.
- (c) Which of the variables is statistically significant?
- (d) Does your finding from part (c) imply that the return on the S&P 500 is predictable? Explain.

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