題號： 106

※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1. (5%) (10%) Let $X \sim NegativeBinomial(r, p)$. State the necessary conditions so that X will converge to a Poisson distribution. Under the given conditions, show that the convergence property holds.

2. (15%) Let $X_1, \ldots, X_n$ be a random sample from the probability density function

$$f_X(x|\theta) = \theta x^{\theta-1}, \ \ 0 < x < 1, \ \ 0 < \theta < \infty.$$

Show that the variance of the the uniformly minimum variance unbiased estimator of θ cannot attain the Cramér-Rao lower bound.

3. (10%) Let $X|Y = y \sim Binomial(y, p)$, $Y|\Lambda = \lambda \sim Poisson(\lambda)$, and $\Lambda \sim Exponential(\beta)$. Compute the variance of X.

4. (15%) Let X and Y be continuous random variables with the corresponding distributions $F_X(x)$ and $F_Y(y)$. Derive the distribution of Y conditioning on $X - Y = 0$.

5. (10%) Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$. Find a function, say $g(\cdot)$, of S_n^2 such that $E[g(S_n^2)] = \sigma^p$ for $(n + p) > 1$.

6. (5%) (15%) State and show the Neyman-Pearson lemma.

7. (15%) Show that a uniformly most powerful level α test is an unbiased test.

試題隨卷繳回