

題號：315

科目：統計學(E)

節次：2

國立臺灣大學 110 學年度碩士班招生考試試題

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1.(15%) When a machine that is used to make bolts at a Steel Corporation is working properly, the mean length of these bolts is 2.5 inches. However, from time to time this machine falls out of alignment and produces bolts that have a mean length of either less than 2.5 inches or more than 2.5 inches. When this happens, the process is stopped and the machine is adjusted. To check whether or not the machine is producing bolts with a mean length of 2.5 inches, the quality control department at the company takes a sample of bolts each week and makes a test of hypothesis.

(1) One such random sample of 49 bolts produced a mean length of 2.49 inches and a standard deviation of 0.021 inches. Using the 5% significance level, can we conclude that the machine needs to be adjusted? (7%)

(2) Suppose the null hypothesis stated in that example is false and the true mean length of all bolts produced by the machine at the time of the selection of the sample was 2.498 inches. What is the probability of making a Type II error if $\alpha=0.05$? What is the power of the test? (8%)

2.(10%) According to the Census Bureau, the mean monthly salary of people with a professional degree was \$4,961 in 2020. Assume that this result holds true for the 2020 population of all people with a professional degree. A random sample of 400 people with a professional degree taken recently showed that their mean monthly salary is \$5,067 with a standard deviation of \$985.

(1) Find the p -value for the test of hypothesis with the alternative hypothesis that the current mean monthly salary of all people with a professional degree is greater than \$4,961. (8%)

(2) If $\alpha=0.01$, based on the p -value calculated in part (1), would you reject the null hypothesis? (2%)

3.(25%) The following table was obtained form the regression analysis.

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 D_1 + \epsilon$$

PREDICTOR	COEF	STDEV	t-RATIO
CONSTANT	46.36	(A)	3.76
x_1	-0.05201	(B)	-3.04
x_2	0.09208	(C)	3.12
x_3	-0.001704	(D)	-0.19
D_1	1.032	(E)	1.02
S = (F)	R-SQ = (G)	R-SQ(ADJ) = 99.6%	
ANALYSIS OF VARIANCE			
SOURCE	DF	SS	MS
REGRESSION	(H)	(I)	(J)
ERROR	9	7.25	(K)
TOTAL	13	2918.00	(L)

Please answer the following questions.

- (1) Fill the blanks (A)-(L). (6%)
- (2) Write the estimated regression equation. (1%)
- (3) Explain the meaning of b_2 obtained by estimating the given regression model. (2%)
- (4) What are the values of the standard deviation of errors, the coefficient of multiple determination, the adjusted coefficient of multiple determination, SST, SSR, SSE, MSR, and MSE? (3%)
- (5) Write the values of the standard deviation, the observed value of t for the estimated coefficient b_4 . (1%)

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- (6) What is the predicted value of y for $x_1=210$, $x_2=195$, $x_3=260$, and $D_1=1$? (1%)
(7) What is the point estimate of the expected (mean) value of y for all elements given that $x_1=290$, $x_2=230$, $x_3=195$, and $D_1=0$? (1%)
(8) Determine a 99% confidence interval for the coefficient of x_1 ? (3%)
(9) Using the 2.5% significance level, can you conclude that the coefficient of x_2 is positive? (3%)
(10) Using the 5% significance level, test whether or not the coefficients of all independent variables in the population regression model are equal to zero. (4%)

4.(10%) Find the moment generating function corresponding to the following probability density functions:

(1) $f(x) = \frac{2x}{c^2}$, $0 < x < c$, where $c > 0$. (5%)

(2) $f(x) = \frac{1}{2} \exp(-|x - \alpha|)$, $-\infty < x < \infty$, where $-\infty < \alpha < \infty$. (5%)

5.(15%) Let X and Y be two independent random variables that follow a Poisson distribution with rate parameter θ and λ , respectively, i.e., $X \sim \text{Poisson}(\theta)$ and $Y \sim \text{Poisson}(\lambda)$. Show that the distribution of $X/(X+Y)$ is binomial with success probability $\theta/(\theta+\lambda)$.

Note: the probability density function of a Poisson distribution with rate parameter γ is

$$f(Z=z) = \frac{\gamma^z e^{-\gamma}}{z!}.$$

6.(25%) Suppose the distribution of Y , conditional on $X=x$, is normal with mean x and variance x^2 and that the marginal distribution of X is uniform (0,1).

(1) Find $E[Y]$, $Var[Y]$, and $Cov(X, Y)$. (15%)

(2) Prove that Y/X and X are independent. (10%)

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Standard Normal Distribution Table

The entries in the table give the areas under the standard normal curve from 0 to z .



z		Area in the Right Tail under the t Distribution Curve									
		.10	.05	.025	.01	.005	.001				
	df										
0.0	.00	.00	.01	.03	.03	.04	.05	.06	.07	.08	.09
0.1	.098	.0458	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753	.0799
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	.1179
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	.1554
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	.1915
0.5	.1915	.1959	.1995	.2019	.2054	.2088	.2123	.2157	.2190	.2224	.2257
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549	.2580
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	.2881
0.8	.2881	.2910	.2939	.2955	.2975	.3023	.3051	.3078	.3106	.3133	.3162
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	.3415
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	.3645
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	.3850
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	.4032
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	.4192
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	.4332
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	.4455
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	.4554
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	.4642
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	.4715
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	.4773
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	.4822
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	.4862
2.2	.4861	.4868	.4872	.4877	.4881	.4887	.4893	.4899	.4905	.4910	.4915
2.3	.4903	.4906	.4909	.4914	.4916	.4919	.4921	.4924	.4927	.4930	.4933
2.4	.4913	.4920	.4922	.4925	.4927	.4929	.4931	.4933	.4936	.4939	.4943
2.5	.4933	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4953	.4955
2.6	.4953	.4956	.4959	.4962	.4965	.4968	.4971	.4974	.4977	.4980	.4983
2.7	.4965	.4966	.4968	.4969	.4971	.4973	.4975	.4977	.4979	.4981	.4983
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4980	.4981	.4982	.4983
2.9	.4981	.4982	.4982	.4983	.4984	.4985	.4985	.4986	.4986	.4987	.4987
3.0	.4987	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4990	.4990	.4990

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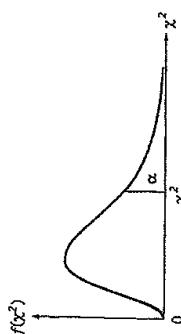
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Critical Values of χ^2 

Degrees of Freedom	$\chi^2_{0.95}$	$\chi^2_{0.90}$	$\chi^2_{0.75}$	$\chi^2_{0.50}$	$\chi^2_{0.30}$	Degrees of Freedom	$\chi^2_{0.10}$	$\chi^2_{0.05}$	$\chi^2_{0.01}$
1	0.000393	0.001571	0.009821	0.039321	.0157008	1	2.70554	3.84146	5.02389
2	.010251	.0201007	.0506356	.102587	.210720	2	4.60517	5.99147	7.37776
3	.0717212	.114832	.215795	.351846	.584375	3	6.25139	7.81473	9.36840
4	.206990	.297110	.484419	.710721	1.063623	4	7.7944	9.48773	11.14433
5	.411740	.554300	.831211	1.145476	1.61031	5	9.23635	11.0705	12.8325
6	.675727	.872085	1.237347	1.65539	2.20413	6	10.6446	12.5916	14.4494
7	.982625	1.239043	1.688987	2.16735	2.83311	7	12.0170	14.0671	16.0128
8	1.344419	1.646482	2.17973	2.73264	3.48954	8	13.3616	15.3073	17.5346
9	1.734926	2.087912	2.70039	3.32511	4.16816	9	14.6837	16.9190	19.0228
10	2.15585	2.55821	3.24697	3.94030	4.86518	10	15.9871	18.3070	20.4831
11	2.60321	3.05347	3.81575	4.57481	5.57779	11	17.2750	19.6751	21.9700
12	3.07382	3.57056	4.40379	5.22603	6.30380	12	18.5494	21.0261	23.3367
13	3.56503	4.10691	5.00874	5.89186	7.04150	13	19.8119	22.3621	24.7256
14	4.07468	4.66043	5.62872	6.57063	7.78953	14	21.0642	23.6848	26.1190
15	4.60094	5.22935	6.26214	7.26094	8.54675	15	22.3072	24.9958	27.5079
16	5.14224	5.81221	6.90766	7.96164	9.31223	16	23.5418	26.2962	28.8454
17	5.69724	6.40776	7.564418	8.67176	10.0852	17	24.7690	27.5871	30.1910
18	6.25481	7.01491	8.23075	9.39046	10.8649	18	25.9894	28.8693	31.5764
19	6.84398	7.63273	8.90655	10.1170	11.6509	19	27.2036	30.1435	32.8523
20	7.43386	8.26040	9.59083	10.8508	12.4426	20	28.4120	31.4104	34.1962
21	8.03366	8.89720	10.28293	11.5913	13.2396	21	29.6151	32.6705	35.4789
22	8.64272	9.54249	10.9823	12.3380	14.0415	22	30.8133	33.9244	36.7807
23	9.26042	10.19567	11.6885	13.0905	14.8479	23	32.0669	35.1725	38.8053
24	9.88623	10.8564	12.4011	13.8484	15.6587	24	33.1963	36.4151	39.3441
25	10.51197	11.5240	13.1197	14.6114	16.4734	25	34.3816	37.6525	40.6465
26	11.1603	12.1981	13.8439	15.3791	17.2919	26	35.5631	38.8852	41.9232
27	11.8076	12.8786	14.5733	16.1513	18.1138	27	36.7412	40.1133	43.1944
28	12.4613	13.5648	15.3079	16.9279	18.9392	28	37.9159	41.3372	44.4607
29	13.1211	14.2565	16.6471	17.7083	19.7677	29	39.0875	42.5569	45.7222
30	13.7867	14.9535	16.5908	18.4926	20.5992	30	40.2560	43.7729	46.9792
40	20.7065	22.1643	24.4331	26.5093	29.0505	40	51.8050	55.7585	59.3417
50	27.9907	29.7067	32.3574	34.7642	37.6886	50	63.1671	67.5048	71.4202
60	35.3346	37.4848	40.4817	43.1879	46.4589	60	74.3970	79.0819	83.2976
70	43.2752	45.4418	48.7576	51.7393	55.3290	70	85.5271	90.5312	95.0231
80	51.1720	53.5400	57.1582	60.3915	64.2278	80	96.5782	101.879	106.6229
90	59.1963	61.7541	65.6466	69.1260	73.2912	90	107.565	113.145	118.136
100	67.3276	70.0648	74.2219	77.9295	82.3581	100	118.498	124.342	129.561

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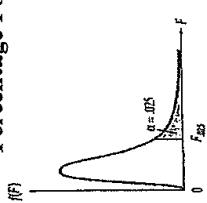
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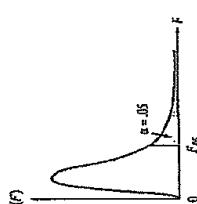
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Percentage Points of the F-distribution, $\alpha = 0.025$ 

v_1	DENOMINATOR DEGREES OF FREEDOM									v_2
	1	2	3	4	5	6	7	8	9	
1	49.8	79.5	36.2	89.6	92.8	93.1	94.2	95.7	96.3	1
2	36.5	39.0	32.5	39.3	39.26	39.37	39.45	39.53	39.61	2
3	17.4	16.04	15.44	14.88	14.73	14.62	14.54	14.47	14.41	3
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	4
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	5
6	8.81	7.26	6.60	6.23	5.93	5.69	5.52	5.40	5.29	6
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	7
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.45	4.36	8
9	7.21	5.71	5.05	4.72	4.46	4.22	4.01	3.83	3.67	9
10	6.94	5.46	4.83	4.47	4.24	3.97	3.65	3.45	3.27	10
11	6.72	5.26	4.63	4.28	4.04	3.88	3.66	3.45	3.27	11
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	12
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	13
14	6.30	4.86	4.24	3.90	3.66	3.50	3.38	3.29	3.21	14
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	15
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	16
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.92	17
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	18
19	5.90	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	19
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	20
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	21
22	5.79	4.38	3.76	3.44	3.22	3.05	2.93	2.84	2.76	22
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	23
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	24
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	25
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	26
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	27
28	5.61	4.22	3.63	3.29	3.06	2.90	2.81	2.71	2.63	28
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	29
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.67	2.57	30
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	40
60	5.29	3.93	3.34	3.01	2.79	2.65	2.51	2.41	2.33	60
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	120
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	∞

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DENOMINATOR DEGREES OF FREEDOM									
		NUMERATOR DEGREES OF FREEDOM							
v_1	v_2	1	2	3	4	5	6	7	8
1	16.14	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	191.06	191.16	191.25	191.30	191.33	191.35	191.37	191.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.39	6.20	6.16	6.09	6.04	6.00	5.99
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.59	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.52	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.86	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	3.63	3.16	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.65	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.85	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.80	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
31	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
32	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
33	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
34	3.84	3.00	2.60	2.37	2.21	2.01	1.94	1.88	1.82

DEGREES OF FREEDOM									
DEMONSTRATOR DEGREES OF FREEDOM									
DENOMINATOR DEGREES OF FREEDOM									
ν_1	ν_2	ν_3	ν_4	ν_5	ν_6	ν_7	ν_8	ν_9	ν_{10}
1	241.9	243.0	245.0	248.0	251.1	251.1	252.2	253.3	254.3
2	19.4	19.45	19.65	19.65	19.46	19.47	19.48	19.49	19.50
3	8.77	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.53
4	5.96	5.96	5.86	5.86	5.75	5.75	5.72	5.69	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.35
6	4.05	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.23
8	3.25	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.40
12	2.75	2.70	2.66	2.62	2.56	2.51	2.47	2.43	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97
19	2.33	2.23	2.23	2.16	2.11	2.07	2.02	1.98	1.93
20	2.25	2.20	2.20	2.12	2.08	2.04	1.99	1.95	1.90
21	2.23	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87
22	2.20	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84
23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75
27	2.20	2.13	2.06	1.97	1.93	1.89	1.84	1.79	1.73
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70
30	2.15	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.69
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58
60	1.92	1.84	1.75	1.67	1.65	1.59	1.53	1.47	1.39
100	1.81	1.73	1.66	1.61	1.55	1.50	1.43	1.35	1.25
200	1.75	1.67	1.60	1.53	1.47	1.41	1.35	1.22	1.10

試題隨卷繳回