

※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

Ordinary Differential Equations

1. Set

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}, \quad (0.1)$$

and $X(t) = (x_1(t), x_2(t))^T$.

(a) (10 points) Calculate $e^{tA} := I + \sum_{j=1}^{\infty} \frac{1}{j!} t^j A^j$, where I is the two by two identity matrix.

(b) (10 points) Solve the differential system

$$\begin{cases} x_1'(t) = x_1(t) + x_2(t) + e^t, \\ x_2'(t) = 2x_2(t), \end{cases} \quad (0.2)$$

with initial condition $X(0) = (1, 0)^T$.

2. (20 points) Solve the differential equation

$$x'''(t) + x''(t) - 16x'(t) - 16x(t) = 0,$$

with initial condition $x(0) = x'(0) = 0, x''(0) = 1$.

3. (20 points) Solve the differential equation

$$x'(t) + \cos tx(t) = \cos t \quad (0.3)$$

with initial condition $x(0) = 0$.

4. Consider the differential equation

$$x'(t) + 2x(t) = \sin t. \quad (0.4)$$

Let $\phi(t)$ be the periodic solution of (0.4).

(a) (10 points) Find the value of $\phi(\pi)$.

(b) (10 points) Let $X(t)$ be any other solution of (0.4). Show that

$$\lim_{t \rightarrow \infty} |X(t) - \phi(t)| = 0.$$

5. (20 points) Solve the differential equation

$$t^2 y''(t) + t y'(t) - y(t) = 0$$

for $t \geq 1$ with $y(1) = 2$ and $y'(1) = 0$.

試題隨卷繳回