1. (20%). Consider a nonhomogeneous first-order difference equation that takes the form

$$r_n = ar_{n-1} + c,$$

where a and c are constants. The equation can be expressed in the matrix form of

$$s_n = As_{n-1}$$

where

$$s_n = \begin{bmatrix} 1 \\ r_n \end{bmatrix}.$$

(a). (3%). Find A.

(b). (3%). Show that $s_n = A^n s_0$.

(c). (8%). Find the eigenvalues and corresponding eigenvectors of A.

(d). (6%). Find r_n (in terms of a, c, and r_0).

2. (10%). Consider the matrix of

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & c \\ 2 & 1 & 7 \end{bmatrix}.$$

(a). (5%). For what scalar c, A is not invertible?

(b). (5%). Assume A is invertible (i.e., $c \notin$ what calculated in (a)). Find the determinant of A^{-1}.

3. (30%). Let $y = y(x)$ be a function of the variable x.

(a). (2%). $\frac{d}{dx} e^x =$?

(b). (3%). Solve $y' + y = 0$.

(c). (10%). Solve the initial value problem $y'' + y' - 2y = 0$; $y(0) = 4$, $y'(0) = -5$.

(d). (15%). Solve the initial value problem $y'' + 3y' + 2.25y = -10e^{-1.5x}$; $y(0) = 1$, $y'(0) = 0$.

4. (40%). Consider a 1D wave equation

$$u_{tt} = u_{xx}, \ 0 < x < 1, \ t > 0.$$

(a). (20%). Find a solution that satisfies the boundary conditions $u(0,t) = 0$ and $u(1,t) = 0$ for $t > 0$, and the initial conditions $u(x,0) = \sin \pi x$ and $u_t(x,0) = 0$ for $0 < x < 1$.

(b). (20%). Find a solution that satisfies the boundary conditions $u(0,t) = 0$ and $u(1,t) = \sin \omega t$ for $t > 0$, and the initial conditions $u(x,0) = 0$ and $u_t(x,0) = 0$ for $0 < x < 1$.

試題隨卷繳回