※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1. (25%) Show that the equation is exact, and obtain its general solution. Also, find the particular solution corresponding to the given initial condition as well.

$$4\cos 2u\,du - e^{-5v}\,dv = 0;\quad v(0) = -6$$

2. (20%) Determine the equation of the phase trajectories for the given system, and sketch several representative trajectories. Use arrows to indicate the direction of movement along those trajectories.

(a) $x' = y,\quad y' = -x$

(b) $x' = xy,\quad y' = -x^2$

3. (25%) Determine whether the following set is LI (Linear independence) or LD (Linear Dependence). If it is LD, then give a linear relation among the vectors.

(a) $(1,3,2,0), (4,1,-2,-2), (0,2,0,3), (4,7,1,2)$

(b) $(2,0,1,-1,0), (1,2,0,3,1), (4,-4,3,-9,-2)$

4. (20%) It is known that the $n\times n$ tridiagonal matrix

$$\mathbf{A} = \begin{bmatrix} b & c & 0 & 0 & & \cdots & 0 \\ a & b & c & 0 & & & \vdots \\ 0 & a & b & c & & & \\ & & & & \ddots & & \vdots \\ \vdots & & & & a & b & c \\ 0 & \cdots & & & \cdots\ 0 & a & b \end{bmatrix}$$

has eigenvalues

$$\lambda_j = b + 2\sqrt{ac}\cos\frac{j\pi}{n+1} \qquad (1)$$

for $j = 1,2,\ldots,n$. (**A** is called tridiagonal because all elements are zero except for those on the main diagonal and the two adjacent diagonals.)

Verify Equation (1) by calculating the eigenvalues for $n = 1$ and $n = 2$.

5. (10%) (a) Is it possible for a matrix to have no eigenvalues? Explain.

(b) A given eigenvalue can have more than one LI eigenvector. Can a given eigenvector correspond to more than one eigenvalue? Explain.

試題隨卷繳回