1. (20%) Find the general solution of the equation $xy'' + y = 0$

2. (10%) (a)Find the solution of the following wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad 0 < t < \infty, \quad ICs. \begin{cases} u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}$$

(10%) (b)If $f(x) = x$, $g(x) = 0$, $-1 \le x \le 1$, find the solutions of $u(-\frac{1}{2}, \frac{2}{3})$, and $u(\frac{1}{3}, \frac{1}{6})$

3. (20%) If the Laplace transform of a function $y(t)$ is $Y(s) = \frac{2s^2 - s}{(s^2+9)^2}$, Find $y(t)$

4. (20%) If a periodic function $f(x)$ can be expressed by its Fourier series

$$f(x) = a_o + \sum_{n=1}^{\infty} \left[ a_n \cos n\omega_o x + b_n \sin n\omega_o x \right].$$

Prove that $\frac{1}{T} \int_{-T/2}^{T/2} f^2(x)dx = a_o{}^2 + \frac{1}{2} \sum_{n=1}^{\infty} \left[ a_n{}^2 + b_n{}^2 \right]$

where $\omega_o \equiv \frac{2\pi}{T}$; $a_o = \frac{1}{T} \int_{-T/2}^{T/2} f(x)dx$; $T$: period of function $f(x)$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos n\omega_o x \, dx; \quad b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin n\omega_o x \, dx;$$

5. (20%) The data points obtained from the experimental measurement are given as follow

| | | | | | |
|---|---|---|---|---|---|
| $x$: | 1 | 2 | 3 | 4 | 5 |
| $y$: | 2 | 5 | 7 | 8 | 10 |

Determine the constants $a$, $b$, such that $y=ax+b$ fits the points with least square error.

試題隨卷繳回