

1. (10%) Find the general solution of the following differential equation.

$$ydx + (x^2y - 2x)dy = 0$$

2. (10%) Find the general solution of the following differential equation.

$$y'''' - 2y'' + y = e^{-x}$$

3. (a) (15%) Find the general solution of the given differential equation.

$$x^2y'' - 2y = x^{m+2}, \quad x > 0, \quad m + 2 > 0 \quad (m: \text{a constant})$$

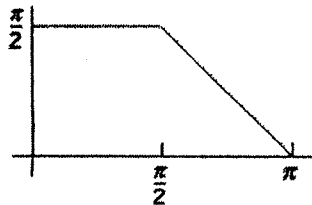
- (b) (5%) Find also the particular solution corresponding to the conditions below.

$$\lim_{x \rightarrow 0} y = 0, \quad y(1) = 0$$

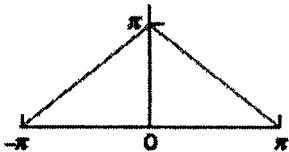
4. (10%) Find the solution to the following initial value problem using the "Laplace transform."

$$ty'' - ty' - y = 0, \quad y(0) = 0, \quad y'(0) = 3$$

5. (5%) Plot the "odd" expansion of the following function f in the range of $|x| \leq 2\pi$.



6. (10%) Find the Fourier series of the given periodic function.



7. (10%) Find the general solutions for the following ODEs.

$$y_1' = 3y_1 - 4y_2$$

$$y_2' = y_1 - y_2$$

8. (10%) $F = [x, y, z^2]$, and S is the surface of a box $|x| \leq 1, |y| \leq 3, 0 \leq z \leq 2$ in space. Evaluate the surface integral $\iint_S F \cdot n \, dA$ by the divergence theorem.

9. (15%) Solve the PDE below:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

IC: $u(x, t=0) = 0$

BCs: $u = 3$ at $x = 0$ for all $t > 0$,

$u = 4$ at $x = 1$ for all $t > 0$.