Write down your answers in order. You should provide all the necessary calculus and reasoning.

1. (10%) Find the horizontal and vertical asymptotes of $y = f(x)$, where

$$f(x) = \frac{\sqrt{x^4+x^3+1} - \sqrt{x^4-x^3+1}}{|x|} + \frac{\ln|x+1|}{x}$$

2. (a) (4%) Show that $f(x) = 1 + x + \int_{-x}^{x} e^{-t^2} dt$ is one-to-one.
   (b) (6%) Let $g(x) = f^{-1}(x)$. Find $g(1)$, $g'(1)$, and $g''(1)$.

3. (10%) Consider $\sum_{i=1}^{1000} \sqrt[3]{i}$ and $\sum_{i=1}^{999} \sqrt[3]{i}$. Use an integral to give $\sum_{i=1}^{1000} \sqrt[3]{i}$ an upper bound and a lower bound.

4. Evaluate the integral or show that it diverges.
   (a) (6%) $\int_1^\infty \frac{\arctan(x)}{x^2} dx$.
   (b) (6%) $\int_0^1 \frac{1}{x\sqrt{1+(\ln x)^2}} dx$.

5. Consider the flow of blood through a blood vessel with cross-section $D = \{(x,y)|x^2+y^2 \le R^2\}$, a small disc with radius $R > 0$. Suppose that the velocity of the blood, v, through each point of the cross-section is $v(x,y) = cP(R^2 - x^2 - y^2)$, where c is a constant and P is the pressure difference between the ends of the vessel. We define the flux of the blood, F, as $F = \iint_D v(x,y) dA$.
   (a) (4%) Derive the formula of F in terms of R and P.
   (b) (8%) Assume that the flux of the blood is constant, and the pressure $P = 4000\ dynes/cm^2$ when $R = 0.008\ cm$. Now the radius of the cross-section decreases at a constant rate $dR/dt = -0.0002\ cm/year$. Find the rate of increase of P when $R = 0.006\ cm$.

6. Consider the ellipse $x^2 + xy + y^2 = 3$.
   (a) (6%) Find the points on which the ellipse obtains the maximum and the minimum y coordinate.
   (b) (6%) Find the equations of the two tangent lines to the ellipse that pass through the point $(4, -2)$.

7. (10%) Use Taylor series to approximate $\int_0^1 \cos(x^5)\, dx$ with error smaller than 10^{-4}.

8. Let

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   (a) (4%) Compute the directional derivative $D_{\vec{u}} f(0,0)$, where $\vec{u} = (\cos\theta, \sin\theta)$. Is $(0,0)$ a critical point of f?
   (b) (8%) Find the maximum and minimum values of f on the unit disc $D = \{(x,y)|x^2 + y^2 \le 1\}$.

9. (12%) Evaluate the integral $\iiint_E z e^{x^2+y^2} dV$, where E is the portion of the unit ball $x^2 + y^2 + z^2 \le 1$ that lies in the first octant and above the cone $z = \sqrt{x^2+y^2}$.